```
> restart:
> with(linalg);
```

[*BlockDiagonal*, *GramSchmidt*, *JordanBlock*, *LUdecomp*, *QRdecomp*, *Wronskian*, *addcol*, *addrow*, *adj*, *adjoint*, *angle*, *augment*, *backsub*, *band*, *basis*, *bezout*, *blockmatrix*, *charmat*, *charpoly*, *cholesky*, *col*, *coldim*, *colspace*, *colspan*, *companion*, *concat*, *cond*, *copyinto*, *crossprod*, *curl*, *definite*, *delcols*, *delrows*, *det*, *diag*, *diverge*, *dotprod*, *eigenvals*, *eigenvalues*, *eigenvectors*, *eigenvects*, *entermatrix*, *equal*, *exponential*, *extend*, *ffgausselim*, *fibonacci*, *forwardsub*, *frobenius*, *gausselim*, *gaussjord*, *geneqns*, *genmatrix*, *grad*, *hadamard*, *hermite*, *hessian*, *hilbert*, *htranspose*, *ihermite*, *indexfunc*, *innerprod*, *intbasis*, *inverse*, *ismith*, *issimilar*, *iszero*, *jacobian*, *jordan*, *kernel*, *laplacian*, *leastsqrs*, *linsolve*, *matadd*, *matrix*, *minor*, *minpoly*, *mulcol*, *mulrow*, *multiply*, *norm*, *normalize*, *nullspace*, *orthog*, *permanent*, *pivot*, *potential*, *randmatrix*, *randvector*, *rank*, *ratform*, *row*, *rowdim*, *rowspace*, *rowspan*, *rref*, *scalarmul*, *singularvals*, *smith*, *stackmatrix*, *submatrix*, *subvector*, *sumbasis*, *swapcol*, *swaprow*, *sylvester*, *toeplitz*, *trace*, *transpose*, *vandermonde*, *vecpotent*, *vectdim*, *vector*, *wronskian*] **(1)**

```
> E7:=diag(1,1,1,1,1,1,1);
```

$$E7 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(2)}$$

```
> A7:=matrix(7,7,[[0.3, 0.1, 0.15, 0.05, 0.03, 0.001, 0.2], [0.05,
  0.4, 0.25, 0.01, 0.02, 0.04, 0.1], [0.01, 0.02, 0.2, 0.05, 0.1,
  0.03, 0.01], [0.02, 0.01, 0.1, 0.3, 0.15, 0.25, 0.03], [0.15,
  0.1, 0.13, 0.11, 0.3, 0.01, 0.02], [0.05, 0.04, 0.07, 0.1, 0.09,
  0.25, 0.04], [0.02, 0.03, 0.01, 0.2, 0.25, 0.15, 0.03]]);
```

(3)

$$A7 := \begin{bmatrix} 0.3 & 0.1 & 0.15 & 0.05 & 0.03 & 0.001 & 0.2 \\ 0.05 & 0.4 & 0.25 & 0.01 & 0.02 & 0.04 & 0.1 \\ 0.01 & 0.02 & 0.2 & 0.05 & 0.1 & 0.03 & 0.01 \\ 0.02 & 0.01 & 0.1 & 0.3 & 0.15 & 0.25 & 0.03 \\ 0.15 & 0.1 & 0.13 & 0.11 & 0.3 & 0.01 & 0.02 \\ 0.05 & 0.04 & 0.07 & 0.1 & 0.09 & 0.25 & 0.04 \\ 0.02 & 0.03 & 0.01 & 0.2 & 0.25 & 0.15 & 0.03 \end{bmatrix} \quad (3)$$

```
> B7:=matadd(E7,-A7);
```

$$B7 := \begin{bmatrix} 0.7 & -0.1 & -0.15 & -0.05 & -0.03 & -0.001 & -0.2 \\ -0.05 & 0.6 & -0.25 & -0.01 & -0.02 & -0.04 & -0.1 \\ -0.01 & -0.02 & 0.8 & -0.05 & -0.1 & -0.03 & -0.01 \\ -0.02 & -0.01 & -0.1 & 0.7 & -0.15 & -0.25 & -0.03 \\ -0.15 & -0.1 & -0.13 & -0.11 & 0.7 & -0.01 & -0.02 \\ -0.05 & -0.04 & -0.07 & -0.1 & -0.09 & 0.75 & -0.04 \\ -0.02 & -0.03 & -0.01 & -0.2 & -0.25 & -0.15 & 0.97 \end{bmatrix} \quad (4)$$

```
> X7:=matrix(7,1,[.20, .35, .20, .47, .45, .21, .48]);
```

$$X7 := \begin{bmatrix} 0.20 \\ 0.35 \\ 0.20 \\ 0.47 \\ 0.45 \\ 0.21 \\ 0.48 \end{bmatrix} \quad (5)$$

```
> sum(X7[i,1],i=1..7);
```

$$2.36 \quad (6)$$

```
> Y7:=multiply(B7,X7);
```

(7)

$$Y7 := \begin{bmatrix} -0.05821 \\ 0.0799 \\ 0.0714 \\ 0.1671 \\ 0.1606 \\ 0.0128 \\ 0.2111 \end{bmatrix} \quad (7)$$

```
> sum(Y7[i,1],i=1..7);
```

$$0.64469 \quad (8)$$

```
> for j from 1 to 7 do b7[j]:=sum(B7[i,j],i=1..7) od;
```

$$b7_1 := 0.40$$

$$b7_2 := 0.30$$

$$b7_3 := 0.09$$

$$b7_4 := 0.18$$

$$b7_5 := 0.06$$

$$b7_6 := 0.269$$

$$b7_7 := 0.57 \quad (9)$$

```
> for j from 1 to 7 do X7[j,1]:=11.94005004*b7[j]/sqrt(sum((b7[i])
  ^2,i=1..7)) od;
```

$$X7_{1,1} := 5.743990324$$

$$X7_{2,1} := 4.307992743$$

$$X7_{3,1} := 1.292397823$$

$$X7_{4,1} := 2.584795646$$

$$X7_{5,1} := 0.8615985487$$

$$X7_{6,1} := 3.862833493$$

$$X7_{7,1} := 8.185186212 \quad (10)$$

```
> sum(X7[i,1],i=1..7);
```

$$26.83879479 \quad (11)$$

```
> sqrt(sum((X7[i,1])^2,i=1..7));
```

$$11.94005004 \quad (12)$$

```
> YY7:=multiply(B7,X7);
```

$$YY7 := \begin{bmatrix} 1.600146467 \\ 0.9583847858 \\ 0.4771819958 \\ 0.1816536944 \\ -1.343950136 \\ 1.683707164 \\ 6.370803269 \end{bmatrix} \quad (13)$$

```
> sum(YY7[i,1],i=1..7);
```

$$9.927927240 \quad (14)$$

```
> sqrt(sum((b7[i])^2,i=1..7));
```

$$0.8314812084 \quad (15)$$

```
> XX7:=matrix(7,1,[x1,x2,x3,x4,x5,x6,x7]);
```

$$XX7 := \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \\ x6 \\ x7 \end{bmatrix} \quad (16)$$

```
> EST7:=matrix(1,7,[1,1,1,1,1,1,1]);
```

$$EST7 := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad (17)$$

```
> multiply(EST7,XX7);
```

$$\begin{bmatrix} x1 + x2 + x3 + x4 + x5 + x6 + x7 \end{bmatrix} \quad (18)$$

```
> multiply(B7,XX7);
```

$$\begin{bmatrix} 0.7\,x1 - 0.1\,x2 - 0.15\,x3 - 0.05\,x4 - 0.03\,x5 - 0.001\,x6 - 0.2\,x7 \\ -0.05\,x1 + 0.6\,x2 - 0.25\,x3 - 0.01\,x4 - 0.02\,x5 - 0.04\,x6 - 0.1\,x7 \\ -0.01\,x1 - 0.02\,x2 + 0.8\,x3 - 0.05\,x4 - 0.1\,x5 - 0.03\,x6 - 0.01\,x7 \\ -0.02\,x1 - 0.01\,x2 - 0.1\,x3 + 0.7\,x4 - 0.15\,x5 - 0.25\,x6 - 0.03\,x7 \\ -0.15\,x1 - 0.1\,x2 - 0.13\,x3 - 0.11\,x4 + 0.7\,x5 - 0.01\,x6 - 0.02\,x7 \\ -0.05\,x1 - 0.04\,x2 - 0.07\,x3 - 0.1\,x4 - 0.09\,x5 + 0.75\,x6 - 0.04\,x7 \\ -0.02\,x1 - 0.03\,x2 - 0.01\,x3 - 0.2\,x4 - 0.25\,x5 - 0.15\,x6 + 0.97\,x7 \end{bmatrix} \quad (19)$$

```
> multiply(EST7,multiply(B7,XX7));
```

$$\begin{bmatrix} 0.40\,x1 + 0.30\,x2 + 0.09\,x3 + 0.18\,x4 + 0.06\,x5 + 0.269\,x6 + 0.57\,x7 \end{bmatrix} \quad (20)$$

```
> with(simplex);
```

[*basis*, *convexhull*, *cterm*, *define_zero*, *display*, *dual*, *feasible*, *maximize*, *minimize*, *pivot*, *pivoteqn*, *pivotvar*, *ratio*, *setup*, *standardize*] (21)

```
> cnsts := {.7*x1-.1*x2-.15*x3-.5e-1*x4-.3e-1*x5-.1e-2*x6-.2*x7>=0,
  -.5e-1*x1+.6*x2-.25*x3-.1e-1*x4-.2e-1*x5-.4e-1*x6-.1*x7>=0,
  -.1e-1*x1-.2e-1*x2+.8*x3-.5e-1*x4-.1*x5-.3e-1*x6-.1e-1*x7>=0,
  -.2e-1*x1-.1e-1*x2-.1*x3+.7*x4-.15*x5-.25*x6-.3e-1*x7>=0,-.15*x1
  -.1*x2-.13*x3-.11*x4+.7*x5-.1e-1*x6-.2e-1*x7>=0,-.5e-1*x1-.4e-1*
  x2-.7e-1*x3-.1*x4-.9e-1*x5+.75*x6-.4e-1*x7>=0,-.2e-1*x1-.3e-1*x2
  -.1e-1*x3-.2*x4-.25*x5-.15*x6+.97*x7>=0,x1+x2+x3+x4+x5+x6+x7=
  22.47795838}:
  obj := .9e-1*x3+.6e-1*x5+.40*x1+.30*x2+.18*x4+.57*x7+.269*x6:
  maximize(obj,cnsts union {x1>=0,x2>=0,x3>=0,x4>=0,x5>=0,x6>=0,
  x7>=0});
```

$\{x1 = 3.739313424, x2 = 2.536118396, x3 = 0.6303503814, x4 = 1.605619587, x5 = 1.858214163, x6 = 1.448949195, x7 = 10.65939323\}$ (22)

```
> subs({ x1 = 3.739313424,x2 = 2.536118395, x3 = .6303503832,  x4 =
  1.605619587,x5 = 1.858214163, x6 = 1.448949195, x7 = 10.65939323}
  ,cnsts);
```

$\{22.47795837 = 22.47795838, 0 \le -6.\ 10^{-10}, 0 \le -4.\ 10^{-10}, 0 \le 0., 0 \le 2.\ 10^{-10}, 0 \le 7.\ 10^{-10}, 0 \le 2.\ 10^{-9}, 0 \le 9.179418268\}$ (23)

```
> subs({ x1 = 3.739313424,x2 = 2.536118395, x3 = .6303503832,  x4 =
  1.605619587,x5 = 1.858214163, x6 = 1.448949195, x7 = 10.65939323}
  ,obj);
```

9.179418273 (24)

```
> subs({x5 = 1.858214163, x1 = 3.739313424, x3 = .6303503832, x2 =
  2.536118395, x4 = 1.605619587, x6 = 1.448949195, x7 =
  10.65939323},sqrt(x1^2+x2^2+x3^2+x4^2+x5^2+x6^2+x7^2));
```

11.94005004 (25)

```
> X7[1,1];X7[2,1];X7[3,1];X7[4,1];X7[5,1];X7[6,1];X7[7,1];
```

5.743990324

4.307992743

1.292397823

2.584795646

0.8615985487

3.862833493

8.185186212 (26)

```
> subs({ x1 = 5.743990324,x2 = 4.307992743, x3 = 1.292397823,  x4 =
  2.584795646,x5 = .8615985487, x6 = 3.862833493, x7 = 8.185186212}
  ,cnsts);
```

$$\{26.83879479 = 22.47795838,\ 0 \le -1.343950136,\ 0 \le 0.1816536944,\ 0 \le 0.4771819958,\ 0 \le 0.9583847858,\ 0 \le 1.600146467,\ 0 \le 1.683707164,\ 0 \le 6.370803269\} \quad (27)$$