Завдання за темою
«Диференціювання алгебраїчних функцій»

Нехай C, α, $a \in R$ $(a>0, a\neq 1)$; $U=U(x)$, $V=V(x)$ – диференційовні функції.

Правила диференціювання:

1. $C' = 0$.
2. $(CU)' = CU'$.
3. $(U \pm V)' = U' \pm V'$.
4. $(UV)' = U'V + UV'$.
5. $\left(\frac{U}{V}\right)' = \frac{U'V - UV'}{V^2}$.
6. $\left(\frac{U}{C}\right)' = \frac{U'}{C}$.
7. $\left(\frac{C}{V}\right)' = -\frac{C}{V^2} \cdot V'$.
8. $[f(U)]' = f'(U) \cdot U'$.

Формули диференціювання:

1. $(U^\alpha)' = \alpha \cdot U^{\alpha-1} \cdot U'$.
2. $(\sqrt{U})' = \frac{1}{2\sqrt{U}} \cdot U'$.
3. $(a^U)' = a^U \cdot \ln a \cdot U'$.
4. $(e^U)' = e^U \cdot U'$.
5. $(\log_a U)' = \frac{1}{U \cdot \ln a} \cdot U'$.
6. $(\ln U)' = \frac{1}{U} \cdot U'$.
7. $(\sin U)' = \cos U \cdot U'$.
8. $(\cos U)' = -\sin U \cdot U'$.
9. $(tgU)' = \frac{1}{\cos^2 U} \cdot U'$.
10. $(ctgU)' = -\frac{1}{\sin^2 U} \cdot U'$.
11. $(\arcsin U)' = \frac{1}{\sqrt{1-U^2}} \cdot U'$.
12. $(\arccos U)' = -\frac{1}{\sqrt{1-U^2}} \cdot U'$.
13. $(arctgU)' = \frac{1}{1+U^2} \cdot U'$.
14. $(arcctgU)' = -\frac{1}{1+U^2} \cdot U'$.

Знайти похідну функцій:

1. $y = \left(x^3 - \frac{3}{x^3} - 2\right)^8$ $y = \frac{-4}{\ln(4x-1)}$; $y = e^{(3^x - 4x)} \cdot x^5$; $y = \frac{e^{4x^2} + 2}{\sqrt{x^2 - 1}}$; $y = 8^{7x-6}$; $y = \ln(2^x - 4)$

2. $y = \left(4x^2 - \frac{3}{\sqrt{x}} + 4\right)^2$; $y = \frac{\sin 2x}{\cos 7x}$; $y = 2^{6x} \cdot \mathrm{tg}3x$; $y = \arcsin \ln 5x$; $y = 5^{6x} \arccos 6x$

3. $y = \left(2x^4 - 3\sqrt{x^5} - 1\right)^4$; $y = 3^{\ln 4x - 2x}$; $y = \frac{\ln 4x}{x^2 - 6x}$; $y = \frac{9}{\sqrt{x^5 + 4x - 1}}$; $y = 3^x \cdot \ln(4x^3 - 5)$;

4. $y = \left(3x - 4\sqrt{x} + 2\right)^5$; $y = 5^{4x - 5\ln x}$; $y = \frac{4x + 7\mathrm{tg}x}{\sqrt{9x^2 - 1}}$; $y = \cos 2x \cdot e^{\sin x}$; $y = \ln \mathrm{arctg}3x$

5. $y=(x^2+2\sqrt[5]{x}+4)^4$; $y=\dfrac{5}{\ln(x^2-1)}$; $y=10^x\cdot\ln(5x+\sqrt{2x}-1)$; $y=\dfrac{e^{3x-6}}{\ln 4x}$; $y=\dfrac{\sqrt{1-7x^5}}{\cos 4x}$

6. $y=(x^2+2\sqrt[5]{x}+4)^4$; $y=\dfrac{x^3-e^x}{\sqrt{4-9x^2}}$; $y=4^{\cos x}\cdot\operatorname{arctg}2x$; $y=\ln\cos 5x$; $y=\dfrac{x^4+\ln x}{\sqrt{4x^2+7}}$.

7. $y=\left(3x-4\sqrt{x}+2\right)^5$; $\boldsymbol{y=e^{3x}\,ln(8x+3)}$; $\boldsymbol{y=5^{x^3-4x}}$; $\boldsymbol{y=\dfrac{6}{ln(4x-7)}}$; $y=\dfrac{\sqrt{6^x-5}}{8x+10x^2}$

8. $y=\left(3x^2+2\sqrt[4]{x}-8\right)^5$; $\boldsymbol{y=11^{2x+5}}$; $y=\dfrac{5}{\ln(x^2-1)}$; $\boldsymbol{y=\dfrac{ln\,8x-8}{(-6x-1)}}$; $\boldsymbol{y=ln\,2x\cdot(x^5-4)}$;

9. $y=\left(2x^4-3\sqrt{x^5}-1\right)^4$; $y=\dfrac{\sqrt{2-x^2}}{\cos 2x}$; $y=5^{\operatorname{arctg}x}\sin 4x$; $y=\ln\arcsin 3x$; $y=e^{\arcsin x}\cos 4x$

10. $y=\left(x^3-4\sqrt{x^3}-2\right)^3$; $\boldsymbol{y=9^{(4x-5\,ln\,2x)}}$; $y=\dfrac{9}{\ln(x^5-1)}$; $\boldsymbol{y=e^{(3-4x)}\cdot x^5}$; $y=\dfrac{6x^2-9^x}{\sqrt{3x^2+7x}}$

11. $y=\left(3x-4\sqrt{x}+2\right)^5$; $y=\dfrac{4x+7\operatorname{tg}x}{\sqrt{9x^2-1}}$; $y=\cos 2x\cdot e^{\sin x}$; $y=\ln\operatorname{arctg}3x$; $y=\dfrac{2x^2-\operatorname{ctg}x}{\sqrt{6x^2-5}}$.

12. $y=\left(6x^3-\dfrac{3}{x^3}+4\right)^2$; $y=\dfrac{\cos 3x}{\sqrt{3x^2+4}}$; $y=4^{\operatorname{tg}x}\arcsin\left(x^2\right)$; $y=\ln\sin 7x$; $y=e^{5x}\cdot(6x^4+7x^2-3)$

13. $y=\left(x^6-\dfrac{3}{x^2}-8\right)^3$; $y=\dfrac{\ln 3x}{\sqrt{3x^2+4}}$; $y=4^{tx}\cdot(x^2+5x-\ln 8)$; $y=\dfrac{-6}{\sqrt{x-3x^9}}$; $\boldsymbol{y=ln(7x^3-9x)}$

14. $y=\left(x^2-\dfrac{1}{x}+5\sqrt{x}\right)^3$; $y=\dfrac{\arcsin 7x}{x^4+e^x}$; $y=e^{\operatorname{tg}x}\ln 3x$; $y=\cos\sqrt[3]{3-x^2}$; $\boldsymbol{y=ln(5x^3+2x)}$

15. $y=\left(6x^3-\dfrac{3}{x^3}+4\right)^2$; $\boldsymbol{y=3\,sin(5x+10)}$; $y=\dfrac{4x-x^5}{\ln(x^3+6)}$; $\boldsymbol{y=\dfrac{6}{\sqrt{5^x-x^3}}}$; $\boldsymbol{y=7^x\cdot ln(x+1)}$;

16. $y=\left(x^5-\sqrt[3]{x}+1\right)^5$; $y=\dfrac{\sqrt{1-4x^2}}{2^x+\operatorname{tg}x}$; $y=e^{\operatorname{ctg}x}\sin 5x$; $y=\sin\ln 6x$; $\boldsymbol{y=8^{x-ln\,4}}$

17. $y=\left(x^2-\dfrac{1}{x}+5\sqrt{x}\right)^3$; $y=\ln(6x-x^6-9)$ $\boldsymbol{y=\dfrac{-7}{\sqrt{5x-3x^2}}}$; $\boldsymbol{y=\dfrac{(4x-7)}{ln\,7x}}$; $\boldsymbol{y=(x^3+2x-7)\,ln\,x}$;

18. $y=\left(3x^5-\dfrac{1}{x^4}+7\right)^3$; $y=\dfrac{x^4+\operatorname{tg}x}{\sqrt{4x^2+7}}$; $y=e^{\arcsin x}\operatorname{ctg}3x$; $y=\operatorname{arctg}\ln 7x$; $\boldsymbol{y=e^{5x}\cdot(6x^4+3)}$

19. $y=\left(x^5-\sqrt[3]{x}+1\right)^5$; $y=\frac{\sqrt{3^x-6x}}{1+9x^3}$; $\boldsymbol{y=\frac{-4}{\ln 9x}}$; $y=\ln\left((4x^3-5)^2\right)$; $\boldsymbol{y=3^x\left(x^5-\ln 6x\right)}$

20. $y=\left(x^3-4\sqrt{x^3}-2\right)^3$; $y=\frac{2^x+\operatorname{ctg}2x}{\sqrt{4+2x^3}}$; $y=e^{\sin x}\arccos 7x$; $y=\operatorname{arctg}\ln 8x$; $y=\ln\operatorname{tg}7x$.

21. $y=\left(3x^5-\frac{1}{x^4}+7\right)^3$; $y=\frac{6}{\sqrt[4]{9x^5-2x}}$; $\boldsymbol{y=\frac{\ln 4x}{x^2-7}}$; $\boldsymbol{y=\sqrt{4x^3+x}}$; $\boldsymbol{y=(3-x^3)\cdot 2^x}$

22. $y=\left(3x^2-\frac{5}{x^3}-2\right)^5$; $y=\frac{\cos 6x}{\sin 3x}$; $y=e^{x^2}\operatorname{tg}5x$; $y=\arcsin\ln 2x$; $\boldsymbol{y=1/\sqrt{\ln 2x-x^7}}$.

23. $y=\left(4x^5-3\sqrt[5]{x^2}-7\right)^3$; $y=\frac{5x-\ln x}{\sqrt{5x^3-1}}$; $y=e^{x^3}\cdot(2x-x^3)$; $\boldsymbol{y=\ln(5x^2+4x-1)}$; $\boldsymbol{y=\frac{-7}{\sqrt{9x-3x^8}}}$;

24. $y=\left(x^4-2\sqrt[3]{x}-1\right)^3$; $y=\frac{\sqrt{3-5x^4}}{e^x-\operatorname{ctg}x}$; $y=2^{\sin x}\arcsin 2x$; $y=\log_2\cos 7x$; $y=\ln 7^x-5x+1$.

25. $y=\left(2-3x+6\sqrt[3]{x^2}\right)^3$; $y=\frac{\sqrt{2-x^2}}{\ln 2x}$; $y=5^x\cdot(4x^2+2x+1)$; $\boldsymbol{y=\ln(3x^2+2)}$; $\boldsymbol{y=\frac{-8}{\ln(x-2)}}$

26. $y=\left(3x^6-\frac{6}{\sqrt{x^3}}+1\right)^8$; $y=\frac{8x^4-5}{\sin 3x}$; $y=2^{\cos x}\ln 2x$; $y=\ln\left(\operatorname{tg}8x-7\right)$; $\boldsymbol{y=\sqrt{\left(4x^3\right)-x^2}}$.

27. $y=\left(3x^2-\frac{5}{x^3}+1\right)^4$; $y=\frac{\sqrt{1-4x^2}}{2^x+\ln x}$; $y=e^{5x}\cdot(5x^3-4x^2+3)$; $\boldsymbol{y=\ln(6x+1)}$; $\boldsymbol{y=\frac{6}{\sqrt{(-3x^3-4)}}}$;

28. $y=\left(9-3\sqrt{x}+\frac{4}{x}\right)^6$; $y=\frac{\ln 4x}{x^2-6x}$; $y=\operatorname{tg}\left(5x\cdot e^{x^2}\right)$; $y=\ln\sin\left(\sqrt{2x}\right)$; $y=3^{2x}\cdot\ln(2x-\frac{9}{x}-1)$.

29. $y=\left(\frac{12}{\sqrt[4]{x^3}}-3x^2+1\right)^4$; $y=\frac{\sqrt{7x-2}}{e^{2x}+4}$; $y=2^{6x}\cdot\sqrt{3x+2}$; $\boldsymbol{y=\ln(5x-5)}$; $\boldsymbol{y=\frac{4}{\ln(9-x^6)}}$;

30. $y=\left(4x^2-\frac{3}{\sqrt{x}}+4\right)^2$; $\boldsymbol{y=\sqrt{\left(x^5-3x\right)}}$; $\boldsymbol{y=\frac{\ln 6x-9}{x+7x^6}}$; $\boldsymbol{y=9^{x-\ln(6x+7)}}$; $\boldsymbol{y=\frac{3}{\sqrt{7+\ln x}}}$; $y=\operatorname{arctg}\ln 5x+8x$