Translation of scientific and technical texts on speciality from a foreign language (Topic 4)

Questions 10 to 25 have several different formats. Unless otherwise directed, select a single answer choice. For Numeric Entry questions, follow the instructions below.

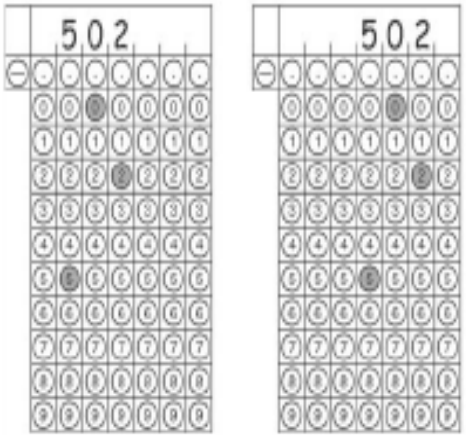
Numeric Entry Questions

To answer these questions, enter a number by filling in circles in a grid.

- Your answer may be an integer, a decimal, or a fraction, and it may be negative.
- Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Fractions do **not** need to be reduced to lowest terms, though you may need to reduce your fraction to fit in the grid.
- Enter the exact answer unless the question asks you to round your answer.
- If a question asks for a fraction, the grid will have a built-in division slash (/). Otherwise, the grid will have a decimal point available.
- Start your answer in any column, space permitting. Fill in no more than one circle in any column of the grid. Columns not needed should be left blank.
- Write your answer in the boxes at the top of the grid and fill in the corresponding circles. **You will receive credit only if the circles are filled in correctly, regardless of the number written in the boxes at the top.**

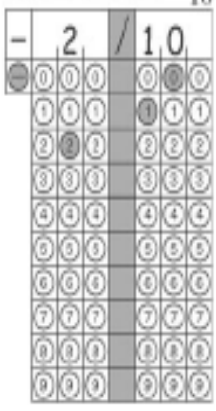
Examples of acceptable ways to use the grid:

Integer answer: 502 (either position is correct)



Decimal answer: −4.13

Fraction answer: $-\frac{2}{10}$



10. Working at their respective constant rates, machine I makes 240 copies in 8 minutes and machine II makes 240 copies in 5 minutes. At these rates, how many more copies does machine II make in 4 minutes than machine I makes in 6 minutes?

(A) 10

(B) 12

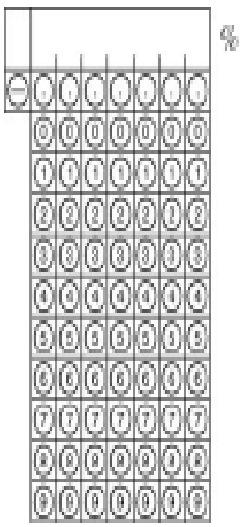
(C) 15

(D) 20

(E) 24

For the following question, use the grid to enter your answer.

11. Among the people attending a convention in Europe, 32 percent traveled from Asia and 45 percent of those who traveled from Asia are women. What percent of the people at the convention are women who traveled from Asia?



12. In the xy-plane, points R and S have coordinates $(-2, 1)$ and $(4, -7)$, respectively. If point P is the midpoint of line segment RS, what are the coordinates of point P ?

(A) $(-1, -3)$

(B) $(1, -4)$

(C) $(1, -3)$

(D) $(2, -4)$

(E) $(3, -4)$

13. Steve's property tax is \$140 less than Patricia's property tax. If Steve's property tax is \$1,960, then Steve's property tax is what percent less than Patricia's property tax, to the nearest 0.1 percent?

(A) 6.7%

(B) 7.1%

(C) 7.5%

(D) 7.9%

(E) 8.3%

14. A base of a triangle has length b, the altitude corresponding to the base has length h, and $b = 2h$. Which of the following expresses the area of the triangle, in terms of h ?

(A) $\frac{1}{2}h^2$

(B) $\frac{3}{4}h^2$

(C) h^2

(D) $\frac{3}{2}h^2$

(E) $2h^2$

15. How many different two-digit positive integers are there in which the tens digit is greater than 6 and the units digit is less than 4 ?

(A) 7

(B) 9

(C) 10

(D) 12

(E) 24

For the following question, select all the answer choices that apply.

16. Chris entered a number in his calculator and erroneously multiplied the number by 2,073 instead of 2.073, getting an incorrect product. Which of the following is a single operation that Chris could perform on his calculator to correct the error?

Indicate all such operations.

[A] Multiply the incorrect product by 0.001

[B] Divide the incorrect product by 0.001

[C] Multiply the incorrect product by 1,000

[D] Divide the incorrect product by 1,000

Translation of scientific texts on speciality into a foreign language (Topic 6)

Task 1. Translate into English

Задача 1. Підприємець придбав два пристрої для виготовлення м’якого морозива. Продуктивність праці першого пристрою 20 кг морозива щогодини, а другого — 30 кг морозива щогодини. За скільки годин підприємець зможе виготовити 100 кг морозива, якщо обидва пристрої працюватимуть одночасно?

Задача 2. Юрій фарбує щогодини 3 м паркану, а Іван — 5 м. На скільки метрів паркану більше пофарбує Іван, якщо робітники працюватимуть разом 6 годин?

Задача 3. Троє робітників за 5 днів викопали 120 м траншеї. Скільки метрів траншеї викопав один робітник за 3 дні, якщо робітники працювали з однаковою продуктивністю?

Задача 4. Один робітник за 5 днів викопав 25 м траншеї, а інший за 4 дні — 24 м. Скільки метрів траншеї робітники викопають за 3 дні, якщо працюватимуть разом із тією самою продуктивністю?

Задача 5. Двоє робітників можуть виконати завдання, працюючи разом, за 2 *дні*. За скільки днів може виконати це завдання кожний робітник, працюючи самостійно, якщо одному з них для виконання $\frac{1}{3}$ завдання треба на *3 дні* менше, ніж другому для виконання $\frac{2}{3}$ завдання?

Let's compare it with the texts of the previous test tasks and correct the translation if necessary.

Making an abstract of the scientific and technical article in a foreign language

(Topic 9)

Task. ***Read the article. Make a list of keywords, suggest a title for the article, and write an abstract and summary***.

Setting and solving research problems contributes to the development of science and is a means of attracting young researchers to new scientific fields. That is why solving research problems is an important part of teaching mathematics. The introduction of research-based learning into education is specified by the National Doctrine of Education Development of Ukraine in the XXI century and is being thoroughly studied by domestic and foreign scientists (Bulvinska, 2019; Kozak, 2016; Vorozhbit-Gorbatyuk et al.,2021).

In the context of research-based learning technology at the university, students are ideally involved in research conducted by teachers. This understanding of the role of the teacher differs significantly from the vision of the teacher as an experienced researcher who organizes students' projects and research activities or uses problem-based learning technology in the educational process. It is clear that today, a large number of teachers are not ready for the full implementation of research-based learning technology, and this is a task for the future that should be gradually and continuously solved.

Unlike typical tasks, the solution of which is often algorithmic, research tasks require the use of general and special methods of cognition. Of particular importance are tasks that develop such important mental skills as the ability to generalize, identify individual cases, reason by analogy, find similarities and differences, etc. The peculiarity of research problems is the fact that a broad mathematical outlook, ingenuity, and creativity are important for solving them, but, on the other hand, it is precisely in solving research problems that these qualities are formed.

The important tasks of higher education are to develop such professional competences of graduates as creativity and flexibility of thinking, the ability to make decisions in non-standard situations, as well as the formation and development of research competence of higher education students, especially of mathematical specialities. When engaging students majoring in secondary education in research, it is also desirable to develop their ability to organize and effectively manage students' research activities (Babak & Vorozhbit-Gorbatyuk, 2021; Proshkin et al., 2018).

Related to the idea of developing research skills in students is the idea of the need to develop so-called non-linear thinking as a new style of scientific thinking. Thinking in the process of invention, for example, is non-linear, as the intended objects, knowledge, and end results are initially unknown and gradually determined by self-organized personal and collaborative efforts. One of the first works on non-linear thinking was the book (Dobronravova, 1990). This idea was backed by numerous domestic and foreign scientists (Harkki et al., 2021; Kremen, 2005).

When solving research problems, it is necessary to think not only about the relationship between specific concepts but also between different sections of mathematics and other disciplines (STEM education). The students (and teachers) need to learn to cope with uncertainty, improvise, and be prepared to analyze the results, which may not be initially foreseeable. The issues of organizing research activities at different stages of the educational process are discussed at conferences, on the pages of scientific and popular science journals, where you can find recommendations on effective approaches to developing advanced skills and thinking necessary to overcome the complex challenges of the modern world (Ammar et al., 2024; Proshkin & Proshkina, 2016).

Realizing the importance of this issue, proactive teachers share their experience of organizing active learning through clubs, elective courses, and supervising the writing of papers in the Junior Academy of Sciences. Famous scientists take part in various summer schools. The organization of research activities provides a certain degree of convergence between university mathematics and school mathematics. The hypothesis that active learning improves performance in science, engineering, and mathematics is confirmed in the work (Freeman et al., 2014). To do this, 225 studies of student performance in undergraduate science, technology, engineering, and mathematics (STEM) courses in traditional lecture versus active learning were analyzed. The results raise questions about the continued use of traditional lectures and support active learning as a desirable, empirically supported teaching practice. When organizing research activities for pupils or students, a good result can only be expected if the proposed research tasks are interesting for them and can motivate students to solve them. Stimulating student motivation is the key issue for achieving the planned learning outcomes. A good course design is essential to keep students motivated to solve the posed tasks. When using problem-based learning technology, students' independence and the ability to function effectively in teams are important. Both are considered at work (Nor Farida et al., 2012) as the motivating elements. The work (Hrybiuk, 2022) describes the results of an experimental study confirming that the use of computer-oriented methodological systems of natural and mathematical research training provides optimal concentration of educational resources, focus of content, and technologies for preparing students for research work, an increase in motivation and efficiency of students' learning.

Setting the research problems under the study of axiomatic theories

In studying the fundamentals of geometry, the students are introduced to various axiomatic theories of the mathematical structure of Euclidean geometry. The theory based on Gilbert's system of axioms is thoroughly considered. Acquaintance with other systems of axioms of Euclidean geometry (Weyl, Aleksandrov, Kolmogorov) is important for mastering the general issues of the axiomatic method, in particular, for developing skills in proving the consistency, independence, and completeness of axiom systems. It is of considerable didactic and methodological importance to prove the equivalence of different axiomatic theories of Euclidean geometry, for example, the Gilbert and Weyl theories. For the deep understanding of this equivalence, it is important to be able to "find the place" of the particular statement in each axiomatic theory, i.e., to build the sequence of statements whose consequence is this statement. This kind of task can certainly be attributed to research.

For example, let us consider the triangle exterior angle theorem: "the exterior angle of a triangle is greater than every interior angle that is not adjacent to it". This theorem plays a very important role in Gilbert's axiomatic theory. It is the consequence of the first three groups of the axiom system and is used to prove many theorems of absolute geometry (Gilbert, 1948). The question of its place in Weyl's axiomatic theory is quite natural when studying the basics of geometry.

The organization of the study of this issue can be characterized as the implementation of the process of project research activity (Grib'yuk & Yunchik, 2016), because it included the following stages: selection of the problem (due to its importance in Gilbert's axiomatic theory); selection of the search concept, collection and analysis of data (by accumulating the necessary amount of educational material); transformation of ideas into the structure (taking into account the presence of the structure of the partially ordered set in the set of statements of the axiomatic theory).

Here is the sequence of statements obtained as the result of the study, each of which is the consequence of the previous ones and the consequence of which is the theorem on the external angle of a triangle in the axiomatic Weyl theory.

Statement 1. In any triangle ABC the following relation takes place

$$AB - BC < AC < AB + BC.$$

Statement 2. In any triangle ABC the following relation takes place:

1) $BC = BA\cos B + CA\cos C$;

2) $CA = CB\cos C + AB\cos A$;

3) $AB = AC\cos A + BC\cos B$.

Statement 3. The following inequalities are fulfilled for internal angles in any triangle ABC:

1) $\cos A + \cos B + \cos C > 1$;

2) $-\cos A + \cos B + \cos C > -1$.

Statement 4. The sum of the cosines of the two interior angles of any triangle is positive.

Statement 5 (on the exterior angle of the triangle). The exterior angle of a triangle is greater than every interior angle of the triangle that is not adjacent to it.

Proof. Let us demonstrate that the exterior angle α at vertex C of the triangle ABC is greater than angle B. According to statement 4, for the interior angles B and C we obtain $\cos B > -\cos C = \cos(\pi - C) = \cos\alpha$, and consequently $\angle B < \alpha$.

Similar questions can be asked about the triangle angle sum theorem, the Pythagorean theorem, etc.

Examples of the research tasks in the course of algebra and number theory

Let us consider ***the application of the properties of the reciprocal polynomial for the solution of a special type of third-degree equation.***

The concept of the polynomial plays an important role in algebra. Among the various types of polynomials, the irreducible polynomial over a field or ring holds a special place due to its possible applications, in particular, in mathematics and cryptography. Another interesting concept in polynomial theory is the concept of the reciprocal polynomial. This concept is not so widespread, but it also has applications, for example, in communication theory. Finding some effective combinations of the properties of the above types of polynomials can be considered as the research task.

For example, it is known that the solution of the equation

$$ax^3 + bx^2 + c = 0 \quad (1)$$

uses the Cardano method. In order to apply Cardano formulas directly, equation (1) is reduced to the following form

$$y^3 + my + n = 0, \quad (2)$$

using the sequence of the substitutions $m = \frac{b}{a}$, $n = \frac{c}{a}$ and $x = y - \frac{m}{3}$.

Let us consider the research task to find the method of reducing equation (1) to the standard form (2) in a different way, the purpose of which is to familiarize students with the concept of a reciprocal polynomial and its possible applications. Let

us recall that the reciprocal polynomial $f^*(x)$ to the polynomial $f(x)$ can be found with the help of the formula $f^*(x) = x^n f\left(\frac{1}{x}\right)$ where n – the power of the original polynomial. We use the focal object method, which consists in focusing on the properties of the given object, comparing them with the properties of other objects (possibly randomly selected), formulating original ideas and implementing them (note that this scheme often includes other methods, such as the brainstorming method with the qualitative system of questions and analysis of student responses).

The application of these methods led to the following research results. The construction of the reciprocal polynomial for the polynomial in equation (1) immediately transforms this equation to the form (2), without applying the above substitutions. In order to write down the roots of the original equation, we use the property of reciprocal polynomials: the roots of the reciprocal polynomial are inverse to the roots of the original polynomial (Van-der-Varden, 1976). Thus, for the polynomial on the left-hand side of the equation $x^3 + 3x^2 - 20 = 0$ the reciprocal polynomial $f^*(x)$ has the form $f^*(x) = x^3\left(\frac{1}{x^3} + 3\frac{1}{x^2} - 20\right) = -20x^3 + 3x + 1$.

Hence, we obtain the equation $x^3 - \frac{3}{20}x - \frac{1}{20} = 0$, which has the standard form. Let us find the roots using Cardano formulas $x_1^* = \frac{1}{2}$, $x_2^* = -\frac{1}{4} + \frac{\sqrt{15}}{20}i$, $x_3^* = -\frac{1}{4} - \frac{\sqrt{15}}{20}i$. Hence $x_1 = 2$, $x_2 = -\frac{5}{2} - \frac{\sqrt{15}}{2}i$, $x_3 = -\frac{5}{2} + \frac{\sqrt{15}}{2}i$ - are the roots of the original equation.

Let us consider ***the application of the properties of the reciprocal and irreducible polynomials in the preparation of the polynomial for the irreducibility test with the help of the Eisenstein criterion.***

This example confirms the fact that research activity can be made more than episodic, demonstrating to students how the results already obtained form the basis for formulating new research ideas, creating conditions for the implementation of stereotype removal operators as one of the components of the theory of research problem solving (Grib'yuk & Yunchik, 2016).

In order to test the polynomial for irreducibility over the field of rational numbers, the Eisenstein criterion is used. Sometimes, to apply this criterion, the polynomial must be transformed by moving to another variable, for example, using the Horner scheme.

Another way of preparing the polynomial for the application of the Eisenstein criterion arises from the following property of reciprocal polynomials: the polynomial, reciprocal to an irreducible polynomial, is irreducible (Markov & Razmyislovich, 1999). For example, in order to prove the irreducibility of the polynomial, let us find the polynomial that is reciprocal to the original polynomial. We obtain the polynomial $f^*(x) = x^3\left(\frac{7}{x^3} + \frac{14}{x^2} + 1\right) = x^3 + 14x + 7$, for which the application of the Eisenstein criterion leads to the conclusion that it is irreducible over the field of rational numbers.