$$\int \frac{dx}{2x+3} = \frac{1}{2}\ln|2x+3|$$

1. $$\int \frac{dx}{x\cdot\sqrt{x^2+x+1}} = \int \frac{dx}{x\sqrt{x^2\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}} = \int \frac{\frac{dx}{x^2}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}} =$$

$$= \left| \begin{array}{l} \frac{1}{x} = t \\ -\frac{dx}{x^2} = dt \end{array} \right| = -\int \frac{dt}{\sqrt{1+t+t^2}} = -\int \frac{dt}{\sqrt{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}} =$$

$$= -\ln\left|t+\frac{1}{2}+\sqrt{1+t+t^2}\right| + C = -\ln\left|\frac{1}{x}+\frac{1}{2}+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\right| + C$$

2. $$\int \frac{x^3-6x^2+3}{\sqrt{x^2+4x+3}}\,dx = (Ax^2+Bx+C)\sqrt{x^2+4x+3} + \lambda\int\frac{dx}{\sqrt{x^2+4x+3}} \ (=)$$

$$\frac{x^3-6x^2+3}{\sqrt{x^2+4x+3}} = \frac{(2Ax+B)\left(\sqrt{x^2+4x+3}\right)^2}{\sqrt{x^2+4x+3}} + (Ax^2+Bx+C)\cdot\frac{\cancel{2}x+\cancel{4}2}{\cancel{2}\sqrt{x^2+4x+3}} + \frac{\lambda}{\sqrt{x^2+4x+3}}$$

$$x^3-6x^2+3 = (2Ax+B)(x^2+4x+3)+(Ax^2+Bx+C)(x+2)+\lambda$$

$$x^3-6x^2+3 = 2Ax^3+8Ax^2+6Ax+Bx^2+4Bx+3B+$$
$$+Ax^3+2Ax^2+Bx^2+2Bx+Cx+2C+\lambda$$

$$\begin{array}{l} x^3: \\ x^2: \\ x: \\ 1: \end{array} \begin{cases} 1 = 3A \\ -6 = 10A+2B \\ 0 = 6A+6B+C \\ 3 = 3B+2C+\lambda \end{cases}$$

$A = \frac{1}{3}$

$B = \frac{-6-10A}{2} = -3-5A = -3-\frac{5}{3} = -\frac{14}{3}$

$C = -6A-6B = -2+28 = 26$

$\lambda = 3-3B-2C = 3+14-52 = -35$

$$(=) \left(\frac{1}{3}x^2-\frac{14}{3}x+26\right)\sqrt{x^2+4x+3} - 35\int\frac{dx}{\sqrt{\underbrace{x^2+4x+3}_{(x+2)^2-1}}} =$$

$$= \left(\frac{1}{3}x^2-\frac{14}{3}x+26\right)\sqrt{x^2+4x+3} - 35\ln\left|x+2+\sqrt{x^2+4x+3}\right| + C$$

3. $$\int \frac{x-\sqrt{x^2+3x+2}}{x+\sqrt{x^2+3x+2}}\,dx = \left| \begin{array}{l} x^2+3x+2=(x+1)(x+2) \\ \sqrt{(x+1)(x+2)} = t(x+1) \\ \cancel{(x+1)}(x+2) = t^2(x+1)^{\cancel{2}} \end{array} \right.$$

$$x+2 = xt^2+t^2$$
$$2-t^2 = xt^2 - x = x(t^2-1)$$
$$x = \frac{2-t^2}{t^2-1}$$

$$\sqrt{(x+1)(x+2)} = t\left(\frac{2-t^2}{t^2-1}+1\right) = t\cdot\frac{2-\cancel{t^2}+\cancel{t^2}-1}{t^2-1} = \frac{t}{t^2-1}$$

$$dx = \frac{-2t(t^2-1)-(2-t^2)\cdot 2t}{(t^2-1)^2}\,dt = \frac{\cancel{-2t^3}+2t-4t+\cancel{2t^3}}{(t^2-1)^2}\,dt = \frac{-2t\,dt}{(t^2-1)^2}$$

$$\Big| = -2\int \frac{\frac{2-t^2}{\cancel{t^2-1}} - \frac{t}{\cancel{t^2-1}}}{\frac{2-t^2}{\cancel{t^2-1}} + \frac{t}{\cancel{t^2-1}}}\cdot\frac{t\,dt}{(t^2-1)^2} =$$

$$= -2\int \frac{2-t^2-t}{2-t^2+t}\cdot\frac{t\,dt}{(t^2-1)^2} = -2\int\frac{t^2+t-2}{t^2-t-2}\cdot\frac{t\,dt}{(t^2-1)^2} =$$

$$= -2\int \frac{(t+2)\cancel{(t-1)}}{(t-2)(t+1)}\cdot\frac{t\,dt}{(t-1)^{\cancel{2}}(t+1)^2} = -2\int\frac{t(t+2)\,dt}{(t-2)(t-1)(t+1)^3} =$$

$$= -2\left(\frac{8}{27}\int\frac{dt}{t-2} - \frac{3}{8}\int\frac{dt}{t-1} + \frac{17}{216}\int\frac{dt}{t+1} - \frac{5}{36}\int\frac{dt}{(t+1)^2} - \frac{1}{6}\int\frac{dt}{(t+1)^3}\right) =$$

$$= -2\left(\frac{8}{27}\ln|t-2| - \frac{3}{8}\ln|t-1| + \frac{17}{216}\ln|t+1| + \frac{5}{36(t+1)} + \frac{1}{12(t+1)^2}\right) + C,$$

$$x+2 = t^2(x+1)$$
$$t^2 = \frac{x+2}{x+1} \Rightarrow t = \sqrt{\frac{x+2}{x+1}}$$

ge $t = \sqrt{\frac{x+2}{x+1}}$

4. $\int \sqrt[3]{3x - x^3}\,dx = \int x^{1/3}(3-x^2)^{1/3}\,dx =$

$$\boxed{\begin{aligned} a^{\frac{m}{n}} &= \sqrt[n]{a^m} \\ a^{-n} &= \frac{1}{a^n} \\ (a^m)^n &= a^{mn} \end{aligned}}$$

$$= \left| \begin{aligned} & m = \tfrac{1}{3} \qquad \frac{m+1}{n} = \frac{\frac{1}{3}+1}{2} = \frac{4}{3\cdot 2} = \frac{2}{3} \\ & n = 2 \\ & p = \tfrac{1}{3} \qquad \frac{m+1}{n} + p = \frac{2}{3} + \frac{1}{3} = 1 \in \mathbb{Z} \end{aligned} \right.$$

$$\left. \begin{aligned} & 3x^{-2} - 1 = t^3 \qquad && \frac{1}{x^2} = \frac{t^3+1}{3} \\ & 3x^{-2} = t^3 + 1 && \\ & x^{-2} = \frac{t^3+1}{3} && x^2 = \frac{3}{t^3+1} \\ & && x = \sqrt{\frac{3}{t^3+1}} = \sqrt{3}\,(t^3+1)^{-\frac{1}{2}} \\ & && dx = \sqrt{3}\left(-\tfrac{1}{2}\right)(t^3+1)^{-3/2}\cdot 3t^2\,dt \end{aligned} \right| =$$

$$= -\int (\sqrt{3})^{1/3}\,(t^3+1)^{-\frac{1}{6}}\left(3 - \frac{3}{t^3+1}\right)^{1/3}\cdot\frac{\sqrt{3}}{2}\,(t^3+1)^{-3/2}\cdot 3t^2\,dt =$$

$$= -3^{1/6}\cdot\frac{\sqrt{3}}{2}\cdot 3\int (t^3+1)^{-1/6}\left(\frac{3t^3 + \cancel{3} - \cancel{3}}{t^3+1}\right)^{1/3}\cdot(t^3+1)^{-3/2}\cdot t^2\,dt =$$

$$= -3^{1/6}\cdot\frac{\sqrt{3}}{2}\cdot 3\cdot 3^{1/3}\int \frac{t^3}{(t^3+1)^2}\,dt = -\frac{9}{2}\int\frac{t^3}{(t^3+1)^2}\,dt$$

$$-\frac{1}{6} - \frac{1}{3} - \frac{3}{2} = \frac{-1-2-9}{6} = -2$$

$$\frac{1}{6} + \frac{1}{2} + \frac{1}{3} = \frac{1+3+2}{6}$$

$\int \quad = \int \quad = \; . \; | \qquad |= \ldots$

$\int \quad =$

$\int \; ..$