ОДНОРІДНІ ДИФЕРЕНЦІАЛЬНІ РІВНЯННЯ

$$\boxed{y' = f\left(\frac{y}{x}\right)}$$

1) $y' = \frac{y}{x} + \frac{x}{y}$

• $z = \frac{y}{x}; \quad y = zx; \quad y' = z'x + z,$

$z'x + z = z + \frac{1}{z} \quad \Rightarrow \quad \frac{dz}{dx}x = \frac{1}{z}, \quad zdz = \frac{dx}{x} \quad \Rightarrow$

$\Rightarrow \quad \frac{z^2}{2} = \ln|x| + \ln C \quad \Rightarrow \quad \frac{z^2}{2} = \ln|Cx|; \quad \frac{y^2}{2x^2} = \ln|Cx|.$ •

2) $y' = \frac{y}{x} + \sin\frac{y}{x}$

• $\frac{y}{x} = z,$

$z'x + z = z + \sin z \quad \Rightarrow \quad \frac{dz}{dx}x = \sin z, \quad \frac{dz}{\sin z} = \frac{dx}{x} \quad \Rightarrow$

$\Rightarrow \quad \ln\left|tg\frac{z}{2}\right| = \ln|x| + \ln|C| \quad \Rightarrow \quad tg\frac{z}{2} = Cx, \quad tg\frac{y}{2x} = Cx.$ •

3) $y' = \frac{x - y}{x + y}$

• $y' = \frac{1 - \frac{y}{x}}{1 + \frac{y}{x}}, \qquad z = \frac{y}{x},$

$$z'x+z=\frac{1-z}{1+z} \quad \Rightarrow \quad z'x=\frac{1-z-z-z^2}{1+z} \quad \Rightarrow$$

$$\Rightarrow \quad \frac{dz(1+z)}{1-2z-z^2}=\frac{dx}{x} \quad \Rightarrow \quad -\frac{1}{2}\int\frac{d(1-2z-z^2)}{1-2z-z^2}=\int\frac{dx}{x} \quad \Rightarrow$$

$$\Rightarrow \quad -\frac{1}{2}\ln\left|1-2z-z^2\right|=\ln|x|+\ln C \quad \Rightarrow \quad \frac{1}{\sqrt{1-2z-z^2}}=Cx \quad \Rightarrow$$

$$\Rightarrow \quad \frac{1}{\sqrt{1-2\frac{y}{x}-\frac{y^2}{x^2}}}=Cx \bullet$$

4) $y'=\dfrac{x+y}{x-y}$

$\bullet\ y'=\dfrac{1+\frac{y}{x}}{1-\frac{y}{x}}, \qquad z=\dfrac{y}{x},$

$$z'x+z=\frac{1+z}{1-z} \quad \Rightarrow \quad \frac{dz\cdot(1-z)}{1+z-z+z^2}=\frac{dx}{x} \Rightarrow \quad \frac{dz\cdot(1-z)}{1+z^2}=\frac{dx}{x} \quad \Rightarrow$$

$$\Rightarrow \quad arctgz-\frac{1}{2}\ln\left|1+z^2\right|=\ln|x|+C \quad \Rightarrow \quad arctg\frac{y}{x}-\frac{1}{2}\ln\left|1+\frac{y^2}{x^2}\right|=\ln|x|+C. \bullet$$

5) $\left(x^2+xy\right)y'=x\sqrt{x^2-y^2}+xy+y^2 \mid : x^2$

$$\bullet\left(1+\frac{y}{x}\right)y'=\sqrt{1-\frac{y^2}{x^2}}+\frac{y}{x}+\frac{y^2}{x^2}$$

$$z=\frac{y}{x};\quad y'=z'x+z;\quad (1+z)(z'x+z)=\sqrt{1-z^2}+z+z^2;$$

$$z'x+z=\frac{\sqrt{(1-z)(1+z)}+z(1+z)}{1+z}; \qquad \Rightarrow \quad z'x+z=\frac{\sqrt{1-z^2}}{1+z}+z \quad \Rightarrow$$

$$\Rightarrow \qquad \frac{1+z}{\sqrt{1-z^2}}dz = \frac{dx}{x} \qquad \Rightarrow$$

$$\| \int \frac{1+z}{\sqrt{1-z^2}}dz = \int \frac{dz}{\sqrt{1-z^2}} - \frac{1}{2}\int \frac{d\left(1-z^2\right)}{\sqrt{1-z^2}} = \arcsin z - \sqrt{1-z^2} \|$$

$$\arcsin\frac{y}{x} - \sqrt{1-\frac{y^2}{x^2}} = \ln|Cx| \bullet$$

Рівняння вигляду

$$\boxed{y' = f\left(\frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}\right)}$$

6) $y' = \dfrac{2x-y+1}{x-2y+1}$

$\bullet \begin{cases} x = x_1 + m \\ y = y_1 + n \end{cases}$

$$\frac{dy_1}{dx_1} = -\frac{2x_1 + 2m - y_1 - n + 1}{-x_1 - m + 2y_1 + 2n - 1}$$

$$\begin{cases} 2m - n + 1 = 0 \\ -m + 2n - 1 = 0 \end{cases} \qquad \Rightarrow \qquad n = \frac{1}{3};\ \ m = -\frac{1}{3}$$

$$\frac{dy_1}{dx_1} = -\frac{2 - \dfrac{y_1}{x_1}}{2\dfrac{y_1}{x_1} - 1} \qquad z = \frac{y_1}{x_1}$$

$$z'x_1 + z = \frac{z-2}{2z-1} \qquad \Rightarrow \qquad \frac{dz\cdot(2z-1)}{z-2-2z^2+z} = \frac{dx_1}{x_1}$$

$$-\frac{1}{2}\ln\left|-2z^2+2z-2\right| = \ln|x_1| + \ln C$$

$$\frac{1}{\sqrt{-2z^2+2z-2}}=Cx_1\Rightarrow\frac{1}{\sqrt{-2\dfrac{y_1^{\,2}}{x_1^{\,2}}+2\dfrac{y_1}{x_1}-2}}=Cx_1\Rightarrow\frac{1}{\sqrt{-2\left(\dfrac{y-\frac{1}{3}}{x+\frac{1}{3}}\right)^2+2\dfrac{y-\frac{1}{3}}{x+\frac{1}{3}}-2}}=Cx+\frac{1}{3}$$

•

7) $y'=\dfrac{-2x+4y-6}{x+y-3}$

Відповідь. $(y-x-1)^2=C(y-2x)^3$

Домашнє завдання.

1) $xdy-ydx=ydy;$

2) $xy'-y=\sqrt{x^2+y^2}$;

3) $y^2+x^2y'=xyy'$;

4) $y'=e^{\frac{y}{x}}+\dfrac{y}{x}$;

5) $xy'=y\ln\dfrac{y}{x}$;

6) $y'=\dfrac{y}{x}+\dfrac{\varphi\left(\frac{y}{x}\right)}{\varphi'\left(\frac{y}{x}\right)}$.