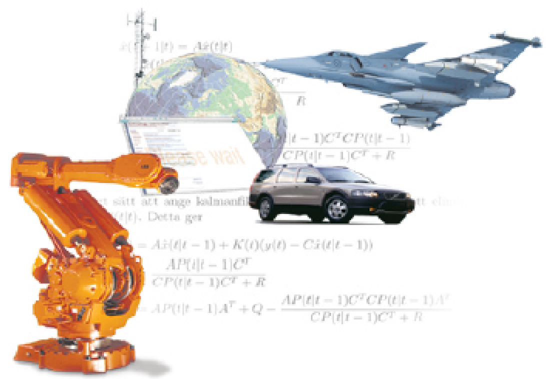
Лаб. 9. **Реалізація методу backstepping для модельних нелінійних систем**

Backstepping

From simple designs to take-off



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Backstepping: From simple designs to take-off

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AUTOMATIC CONTROL
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Example 1 (backstepping)

$$\begin{aligned}\dot{x}_1 &= x_1^2 + x_2\\ \dot{x}_2 &= u\end{aligned}$$

Step 1: $x_{2,d} = -x_1^2 - x_1$

$$V_1 = \tfrac{1}{2}x_1^2$$

$$\dot{V}_1 = x_1\dot{x}_1 = -x_1^2 \le 0$$

if $x_2 = x_{2,d}$

Step 2: $\tilde{x}_2 = x_2 - x_{2,d} = x_2 + x_1^2 + x_1$

$$\begin{cases}\dot{x}_1 = -x_1 + \tilde{x}_2\\ \dot{\tilde{x}}_2 = u + \underbrace{(2x_1 + 1)(-x_1 + \tilde{x}_2)}_{\phi}\end{cases}$$

$$V_2 = \tfrac{1}{2}x_1^2 + \tfrac{1}{2}\tilde{x}_2^2$$

$$\dot{V}_2 = x_1(-x_1 + \tilde{x}_2) + \tilde{x}_2(u + \phi)$$

$$\Rightarrow \quad = -x_1^2 + \tilde{x}_2(x_1 + u + \phi)$$

$$= -x_1^2 - \tilde{x}_2^2 \le 0$$

if $u = -x_1 - \phi - \tilde{x}_2$

Example 1 (feedback linearization)

$$\begin{aligned}\dot{x}_1 &= x_1^2 + x_2\\ \dot{x}_2 &= u\end{aligned}$$

$$y = x_1 = \mathbf{z}_1$$

$$\dot{\mathbf{z}}_1 = x_1^2 + x_2 = \mathbf{z}_2$$

$$\dot{\mathbf{z}}_2 = 2\mathbf{z}_1\mathbf{z}_2 + u$$

$$\Downarrow$$

$u = -2\mathbf{z}_1\mathbf{z}_2 - k_1\mathbf{z}_1 - k_2\mathbf{z}_2$ gives stability

? Which control law should I choose?

! Same control law with $k_1 = k_2 = 2$

Example 1 (backstepping)

$$\dot{x}_1 = x_1^2 + x_2$$
$$\dot{x}_2 = u$$

Step 1: $x_{2,d} = -x_1^2 - x_1$

$$V_1 = \tfrac{1}{2}x_1^2$$

$$\dot{V}_1 = x_1\dot{x}_1 = -x_1^2 \le 0$$

if $x_2 = x_{2,d}$

Step 2: $\tilde{x}_2 = x_2 - x_{2,d} = x_2 + x_1^2 + x_1$

$$\begin{cases} \dot{x}_1 = -x_1 + \tilde{x}_2 \\ \dot{\tilde{x}}_2 = u + (2x_1 + 1)(-x_1 + \tilde{x}_2) \end{cases}$$

ϕ

$$V_2 = \tfrac{1}{2}x_1^2 + \tfrac{1}{2}\tilde{x}_2^2$$

$$\dot{V}_2 = x_1(-x_1 + \tilde{x}_2) + \tilde{x}_2(u + \phi)$$
$$= -x_1^2 + \tilde{x}_2(x_1 + u + \phi)$$
$$= -x_1^2 - \tilde{x}_2^2 \le 0$$

if $u = -x_1 - \phi - \tilde{x}_2$

Example 2 (adaptive backstepping)

$$\dot{x}_1 = x_1^2 + x_2$$
$$\dot{x}_2 = u + \theta x_2^2$$

Step 1: $x_{2,d} = -x_1^2 - x_1$

$$V_1 = \tfrac{1}{2}x_1^2$$

$$\dot{V}_1 = x_1\dot{x}_1 = -x_1^2 \le 0$$

Step 2: $\tilde{x}_2 = x_2 - x_{2,d} = x_2 + x_1^2 + x_1$

$$\begin{cases} \dot{x}_1 = -x_1 + \tilde{x}_2 \\ \dot{\tilde{x}}_2 = u + (2x_1 + 1)(-x_1 + \tilde{x}_2) + \theta x_2^2 \end{cases}$$

$$V_2 = \tfrac{1}{2}x_1^2 + \tfrac{1}{2}\tilde{x}_2^2 + \tfrac{1}{2}(\theta - \hat{\theta})^2$$

$$\dot{V}_2 = -x_1^2 + \tilde{x}_2(x_1 + u + \phi + \theta x_2^2) - (\theta - \hat{\theta})\dot{\hat{\theta}}$$

$u = -x_1 - \phi - \tilde{x}_2 - \hat{\theta}x_2^2$ gives

$$\dot{V}_2 = -x_1^2 - \tilde{x}_2^2 + (\theta - \hat{\theta})\left(\tilde{x}_2 x_2^2 - \dot{\hat{\theta}}\right) \le 0$$

if $\dot{\hat{\theta}} = -\tilde{x}_2 x_2^2$

Завдання для самостійного виконання

Визначити функцію керування u=u(x1,x2), що забезпечує стабілізацію нульового розв'язка системи та перевірити результат

1. $$\begin{aligned}\dot{x}_1 &= x_1^{\,2} + x_2,\\ \dot{x}_2 &= x_1 + u.\end{aligned}$$

2. $$\begin{aligned}\dot{x}_1 &= x_1^{\,2} + x_2,\\ \dot{x}_2 &= x_2^{\,2} + u.\end{aligned}$$