

ОДНОРОДНЫЕ ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ

$$\boxed{y' = f\left(\frac{y}{x}\right)}$$

1) $y' = \frac{y}{x} + \frac{x}{y}$

• $z = \frac{y}{x}; \quad y = zx; \quad y' = z'x + z,$

$$z'x + z = z + \frac{1}{z} \quad \Rightarrow \quad \frac{dz}{dx}x = \frac{1}{z}, \quad z dz = \frac{dx}{x} \quad \Rightarrow$$

$$\Rightarrow \quad \frac{z^2}{2} = \ln|x| + \ln C \quad \Rightarrow \quad \frac{z^2}{2} = \ln|Cx|; \quad \frac{y^2}{2x^2} = \ln|Cx|. \bullet$$

2) $y' = \frac{y}{x} + \sin \frac{y}{x}$

• $\frac{y}{x} = z,$

$$z'x + z = z + \sin z \quad \Rightarrow \quad \frac{dz}{dx}x = \sin z, \quad \frac{dz}{\sin z} = \frac{dx}{x} \quad \Rightarrow$$

$$\Rightarrow \quad \ln \left| \operatorname{tg} \frac{z}{2} \right| = \ln|x| + \ln|C| \quad \Rightarrow \quad \operatorname{tg} \frac{z}{2} = Cx, \quad \operatorname{tg} \frac{y}{2x} = Cx. \bullet$$

3) $y' = \frac{x-y}{x+y}$

• $y' = \frac{1 - \frac{y}{x}}{1 + \frac{y}{x}}, \quad z = \frac{y}{x},$

$$z'x + z = \frac{1-z}{1+z} \quad \Rightarrow \quad z'x = \frac{1-z-z-z^2}{1+z} \quad \Rightarrow$$

$$\Rightarrow \quad \frac{dz(1+z)}{1-2z-z^2} = \frac{dx}{x} \quad \Rightarrow \quad -\frac{1}{2} \int \frac{d(1-2z-z^2)}{1-2z-z^2} = \int \frac{dx}{x} \quad \Rightarrow$$

$$\Rightarrow \quad -\frac{1}{2} \ln|1-2z-z^2| = \ln|x| + \ln C \quad \Rightarrow \quad \frac{1}{\sqrt{1-2z-z^2}} = Cx \quad \Rightarrow$$

$$\Rightarrow \quad \frac{1}{\sqrt{1-2\frac{y}{x}-\frac{y^2}{x^2}}} = Cx \bullet$$

$$4) y' = \frac{x+y}{x-y}$$

$$\bullet y' = \frac{1 + \frac{y}{x}}{1 - \frac{y}{x}}, \quad z = \frac{y}{x}$$

$$z'x + z = \frac{1+z}{1-z} \Rightarrow \frac{dz \cdot (1-z)}{1+z-z+z^2} = \frac{dx}{x} \Rightarrow \frac{dz \cdot (1-z)}{1+z^2} = \frac{dx}{x} \Rightarrow$$

$$\Rightarrow \operatorname{arctg} z - \frac{1}{2} \ln|1+z^2| = \ln|x| + C \Rightarrow \operatorname{arctg} \frac{y}{x} - \frac{1}{2} \ln \left| 1 + \frac{y^2}{x^2} \right| = \ln|x| + C. \bullet$$

$$5) (x^2 + xy)y' = x\sqrt{x^2 - y^2} + xy + y^2 \quad | : x^2$$

$$\bullet \left(1 + \frac{y}{x}\right)y' = \sqrt{1 - \frac{y^2}{x^2}} + \frac{y}{x} + \frac{y^2}{x^2}$$

$$z = \frac{y}{x}; \quad y' = z'x + z; \quad (1+z)(z'x + z) = \sqrt{1-z^2} + z + z^2;$$

$$z'x + z = \frac{\sqrt{(1-z)(1+z)} + z(1+z)}{1+z}; \quad \Rightarrow \quad z'x + z = \frac{\sqrt{1-z^2}}{1+z} + z \Rightarrow$$

$$\Rightarrow \frac{1+z}{\sqrt{1-z^2}} dz = \frac{dx}{x} \Rightarrow$$

$$\| \int \frac{1+z}{\sqrt{1-z^2}} dz = \int \frac{dz}{\sqrt{1-z^2}} - \frac{1}{2} \int \frac{d(1-z^2)}{\sqrt{1-z^2}} = \operatorname{arcsin} z - \sqrt{1-z^2} \|$$

$$\operatorname{arcsin} \frac{y}{x} - \sqrt{1 - \frac{y^2}{x^2}} = \ln|Cx| \bullet$$

$$6) y' = \frac{2x - y + 1}{x - 2y + 1}$$

$$\bullet \begin{cases} x = x_1 + m \\ y = y_1 + n \end{cases}$$

$$\frac{dy_1}{dx_1} = - \frac{2x_1 + 2m - y_1 - n + 1}{-x_1 - m + 2y_1 + 2n - 1}$$

$$\begin{cases} 2m - n + 1 = 0 \\ -m + 2n - 1 = 0 \end{cases} \Rightarrow n = \frac{1}{3}; \quad m = -\frac{1}{3}$$

$$\frac{dy_1}{dx_1} = - \frac{2 - \frac{y_1}{x_1}}{2 \frac{y_1}{x_1} - 1} \quad z = \frac{y_1}{x_1}$$

$$z'x_1 + z = \frac{z-2}{2z-1} \quad \Rightarrow \quad \frac{dz \cdot (2z-1)}{z-2-2z^2+z} = \frac{dx_1}{x_1}$$

$$-\frac{1}{2} \ln |-2z^2 + 2z - 2| = \ln |x_1| + \ln C$$

$$\frac{1}{\sqrt{-2z^2 + 2z - 2}} = Cx_1 \Rightarrow \frac{1}{\sqrt{-2\frac{y_1^2}{x_1^2} + 2\frac{y_1}{x_1} - 2}} = Cx_1 \Rightarrow \frac{1}{\sqrt{-2\left(\frac{y-\frac{1}{3}}{x+\frac{1}{3}}\right)^2 + 2\frac{y-\frac{1}{3}}{x+\frac{1}{3}} - 2}} = Cx + \frac{1}{3}$$

$$7) y' = \frac{-2x + 4y - 6}{x + y - 3}$$

$$\text{ОТВЕТ: } (y - x - 1)^2 = C(y - 2x)^3$$

101. $(x + 2y) dx - x dy = 0.$

102. $(x - y) dx + (x + y) dy = 0.$

103. $(y^2 - 2xy) dx + x^2 dy = 0.$

104. $2x^3 y' = y(2x^2 - y^2).$

105. $y^2 + x^2 y' = x y y'.$

106. $(x^2 + y^2) y' = 2xy.$

107. $x y' - y = x \operatorname{tg} \frac{y}{x}.$

108. $x y' = y - x e^{y/x}.$

109. $x y' - y = (x + y) \ln \frac{x+y}{x}.$

110. $x y' = y \cos \ln \frac{y}{x}.$

111. $(y + \sqrt{xy}) dx = x dy.$

112. $x y' = \sqrt{x^2 - y^2} + y.$

113. $(2x - 4y + 6) dx + (x + y - 3) dy = 0.$

114. $(2x + y + 1) dx - (4x + 2y - 3) dy = 0.$

115. $x - y - 1 + (y - x + 2)y' = 0.$

116. $(x + 4y)y' = 2x + 3y - 5.$

117. $(y + 2) dx = (2x + y - 4) dy.$

118. $y' = 2 \left(\frac{y+2}{x+y-1} \right)^2.$

119. $(y' + 1) \ln \frac{y+x}{x+3} = \frac{y+x}{x+3}.$

120. $y' = \frac{y+2}{x+1} + \operatorname{tg} \frac{y-2x}{x+1}.$

121. $x^3(y' - x) = y^2.$

122. $2x^2y' = y^3 + xy.$

123. $2x dy + (x^2y^4 + 1)y dx = 0.$

124. $y dx + x(2xy + 1) dy = 0.$