ДИФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ С РАЗДЕЛЯЮЩИМИСЯ

1) $xydx + (x+1)dy = 0$

• $xydx = -(x+1)dy \,|:(x+1):y$

$\frac{x}{x+1}dx = -\frac{dy}{y} \quad \Rightarrow \quad \int \frac{x}{x+1}dx = -\int \frac{dy}{y} \quad \Rightarrow$

$\Rightarrow \quad \int \frac{x+1}{x+1}dx - \int \frac{dx}{x+1} = -\int \frac{dy}{y} \quad \Rightarrow \quad x - \ln|x+1| = -\ln|y| + \ln C$ •

2) $(x^2-1)y' + 2xy^2 = 0 \,|: y^2 : (x^2-1)$

• $\frac{y'}{y^2} + \frac{2x}{x^2-1} = 0 \quad \Rightarrow \quad \frac{1}{y^2}\cdot\frac{dy}{dx} + \frac{2x}{x^2-1} = 0 \quad \Rightarrow$

$\Rightarrow \quad \frac{dy}{y^2} = -\frac{2x}{x^2-1}dx \quad \Rightarrow \quad \int \frac{dy}{y^2} = -\int \frac{2x}{x^2-1}dx \quad \Rightarrow$

$\Rightarrow \quad -\frac{1}{y} = -\ln|x^2-1| + \ln C$ •

3) $y' ctgx + y = 2$

• $\frac{dy}{dx}ctgx = 2 - y \,|:(2-y):ctgx$

$\frac{dy}{dx}\cdot\frac{1}{2-y} = \frac{1}{ctgx} \quad \Rightarrow \quad \frac{dy}{2-y} = \frac{dx}{ctgx} \quad \Rightarrow$

$\Rightarrow \quad \int \frac{dy}{2-y} = \int \frac{dx}{ctgx} + C \Rightarrow \int \frac{dx}{ctgx} = \int \frac{\sin x dx}{\cos x} = -\int \frac{d(\cos x)}{\cos x} = -\ln|\cos x| \Rightarrow$

$\Rightarrow \quad -\ln|y-2| = -\ln|\cos x| + C$ •

Уравнения вида

$$\boxed{y' = f(ax+by+c)}$$

Замена $z = ax + by + c$.

4) $y' = \cos(y-x)$

• $z = y - x, \quad \Rightarrow \quad \frac{dz}{dx} = \frac{dy}{dx} - 1$

$\frac{dz}{dx} = \cos z - 1; \quad \Rightarrow \quad \frac{dz}{\cos z - 1} = dx \quad \Rightarrow \quad \int \frac{dz}{\cos z - 1} = \int dx + C;$

$\int \frac{dz}{\cos z - 1} =$

$\| \; tg\frac{z}{2} = t; \quad \cos z = \frac{1-t^2}{1+t^2}; \quad \cos z - 1 = \frac{1-t^2-t^2-1}{t^2+1} = \frac{-2t^2}{t^2+1}; \quad dz = \frac{2dt}{1+t^2} \; \|$

$$=\int\frac{(t^2+1)}{-2t^2}\cdot\frac{2dt}{1+t^2}=-\int\frac{dt}{t^2}=\frac{1}{t}+C=\frac{1}{tg\,\frac{z}{2}}+C=ctg\,\frac{z}{2}+C\,.$$

Тогда

$$ctg\,\frac{z}{2}=x+C \qquad \Rightarrow \qquad x+C=ctg\,\frac{y-x}{2}\ \bullet$$

5) $y'-y=2x-3$

$\bullet$ $y'=2x+y-3$

$2x+y-3=z$ – замена

$$\frac{dz}{dx}=2+\frac{dy}{dx} \qquad \Rightarrow \qquad \frac{dz}{dx}=2+z;\ \Rightarrow \frac{dz}{2+z}=dx; \qquad \Rightarrow$$

$$\Rightarrow \quad \int\frac{dz}{2+z}=\int dx+C; \qquad \Rightarrow \quad \ln|z+2|=x+C \quad \Rightarrow$$

$$\Rightarrow \quad \ln|2x+y-1|=x+C\ \bullet$$

6) $y'=\sqrt{4x+2y-1}$

$\bullet$ $4x+2y-1=z-$ замена

$$\frac{dz}{dx}=4+2\frac{dy}{dx};$$

$$\frac{dz}{dx}=4+2\sqrt{z} \qquad \Rightarrow \qquad \frac{dz}{4+2\sqrt{z}}=dx \qquad \Rightarrow$$

$$\Rightarrow \quad \frac{1}{2}\int\frac{dz}{2+\sqrt{z}}=\int dx$$

$\|\ 2+\sqrt{z}=t$

$$dt=\frac{1}{2}\frac{1}{\sqrt{z}}dz \qquad \Rightarrow \qquad dz=2\sqrt{z}dt=2(t-2)dt\,.$$

Тогда

$$\int\frac{dz}{\sqrt{z}+2}=\int\frac{2(t-2)dt}{t}=2\left(\int\frac{t}{t}dt-2\int\frac{dt}{t}\right)=2(t-2\ln|t|)=$$

$$=2\left(2+\sqrt{z}-2\ln\left|2+\sqrt{z}\right|\right)\ \Big\|$$

$$2+\sqrt{z}-2\ln\left|2+\sqrt{z}\right|=x+C$$

$$x=2+\sqrt{4x+2y-1}-2\ln\left|2+\sqrt{4x+2y-1}\right|-C\ \bullet$$

51. $xy\,\mathrm{d}x + (x+1)\,\mathrm{d}y = 0.$

52. $\sqrt{y^2+1}\,\mathrm{d}x = xy\,\mathrm{d}y.$

53. $(x^2-1)y' + 2xy^2 = 0;\ \ y(0) = 1.$

54. $y'\operatorname{ctg} x + y = 2;\ \ y(x) \to -1$ при $x \to 0.$

55. $y' = 3\sqrt[3]{y^2};\ \ y(2) = 0.$

56. $xy' + y = y^2;\ \ y(1) = 0{,}5.$

57. $2x^2yy' + y^2 = 2.$

58. $y' - xy^2 = 2xy.$

59. $\mathrm{e}^{-s}\left(1 + \frac{\mathrm{d}s}{\mathrm{d}t}\right) = 1.$

60. $z' = 10^{x+z}.$

61. $x\frac{\mathrm{d}x}{\mathrm{d}t} + t = 1.$

62. $y' = \cos(y - x).$

63. $y' - y = 2x - 3.$

64. $(x+2y)y' = 1;\ \ y(0) = -1.$

65. $y' = \sqrt{4x + 2y - 1}.$