Positive observer design for discrete-time positive system with missing data in output

Article · July 2015

DOI: 10.1109/CCDC.2015.7161689

Guoliang Wang
Liaoning ShiHua University

Neurocomputing 168 (2015) 427–434

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Positive observer design for discrete-time positive system with missing data in output ☆

Guoliang Wang [a,*], Boyu Li [a], Qingling Zhang [b,c], Chunyu Yang [d]

[a] School of Information and Control Engineering, Liaoning Shihua University, Fushun 113001, Liaoning, China
[b] Institute of Systems Science, Northeastern University, Shenyang 110004, Liaoning, China
[c] State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Shenyang 110004, Liaoning, China
[d] School of Information and Electrical Engineering, China University of Mining and Technology, Xuzhou 221116, Jiangsu, China

ARTICLE INFO

Article history:
Received 3 January 2015
Received in revised form
8 April 2015
Accepted 21 May 2015
Communicated by Hongyi Li
Available online 10 June 2015

Keywords:
Discrete-time positive system
Positive observer
Missing data
Markov chain

ABSTRACT

This paper is concerned with the positive observer design for a class of discrete-time positive systems with missing data in output. Such a process of missing data is described by a discrete-time Markov chain with two modes. Firstly, a necessary and sufficient condition is proposed to check the positivity of the modeled system. Then, necessary and sufficient conditions depending on some probabilities are developed such that the error system is asymptotically mean stable. Based on the given results, sufficient conditions for the existence of the desired positive observer are provided. Moreover, the obtained results are extended to more general cases that the transition probabilities are uncertain and totally unknown. Finally, numerical examples are used to demonstrate the effectiveness and superiority of the proposed methods.

1. Introduction

There exist many practical examples found in population models, economics, ecology and communication [1–4], whose variables with negative values have no physical meaning. Such systems are commonly modeled as positive systems whose state and output are nonnegative values for any given nonnegative initial states and inputs. Over the past decades, positive systems have been a hot topic in system theory and application. A great many of results on various topics such as positive realization [5], reachability [6,7], positivity analysis [8–11], positive stabilization [12–15], and positive filter [16] have emerged.

As we know, because of technical or economical constraints, it is not easy or expensive to get all the state variables in many practical applications. Then, it is necessary and important to design observers to estimate state variables. Up to now, the observer design problems for various kinds of systems were considered in [17–26]. From these references, it is said that the observer design techniques developed for generally dynamical systems may not be suitable for positive systems. That is because there is often a positive constraint on the observer design of positive systems [27–32]. Moreover, by investigating the above references about the positive observer design, it is claimed that there is no reference to report the problem that the output occupies missing data. However, this phenomenon may be very common in many practically dynamical systems such as [33–38]. A typical application of such systems is networked control systems (NCSs). When the underlying system is closed by the real time networks, because of the effects of networks such as induced delay and packet dropout, it makes the transmitted signal obtained with some probability instead of being totally available or inaccessible, for instance [39–43]. Based on these facts, it is very significative to design observers for systems with data missing. To our best knowledge, very few results are available to design positive observers for positive systems with missing data in output. All the observations motivate the current research.

In this paper, the positive observer design problem of discrete-time positive systems with missing data in output is firstly studied. The main contributions of this paper are summarized as follows: (1) the missing data in output is modeled to be a discrete-time Markov chain having two modes and is considered in the positive observer design of positive systems; (2) necessary and sufficient condition for the positivity of discrete-time system with a stochastic variable is

☆This work was supported by the National Natural Science Foundation of China under Grants 61104066, 61374043 and 61473140, the China Postdoctoral Science Foundation funded project under Grant 2012M521086, the Program for Liaoning Excellent Talents in University under Grant LJQ2013040, the Natural Science Foundation of Liaoning Province under Grant 2014020106, Jiangsu Provincial Natural Science Foundation of China under Grant BK20130205. A preliminary version of this work was first appeared at the 27th Chinese Control and Decision Conference, Qingdao, China
* Corresponding author.
E-mail address: glwang985@163.com (G. Wang).

http://dx.doi.org/10.1016/j.neucom.2015.05.084

presented; (3) necessary and sufficient conditions for checking its asymptotically mean stability are developed to depend on some probabilities, which are essential to design positive observers; (4) several sufficient conditions for designing observers are established, which are convenient to be computed; (5) the given results are further extended to some general cases that such probabilities are uncertain and totally unknown.

An outline of the content of this paper is as follows: In Section 2, we propose a kind of model for positive observer with missing output, where the process of data missing is described by a discrete-time Markov chain. Some essential definitions and a theorem on positivity are also provided in this section. In Section 3, some equivalent statements about the obtained model are given, and several kinds of LMI conditions for the existence of positive observer are established to depend on such probabilities. Moreover, when these probabilities are uncertain and totally unknown respectively, some different existence conditions for the desired observer are developed with their parameters constructed. In Section 4, the effectiveness and superiority of the proposed methods are shown by several examples. Section 5 concludes this paper.

Notation 1. $\mathbb{R}^n$ denotes the n dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ real matrices. $\mathbb{R}_+^{m\times n}$ is the set of all $m\times n$ real matrices with nonnegative entries. $\mathbb{Z}^+$ is the set of positive integer. Set $\mathbb{Z}^+[a,b)$ with $a\in\mathbb{Z}^+$ and $b\in\mathbb{Z}^+$ is defined as $\{k|a\le k<b, \forall k\in\mathbb{Z}^+\}$. A real matrix or vector M is called positive (respectively, strictly positive) and denoted by $M\succcurlyeq 0$ (respectively, $M\succ 0$), if all its components are nonnegative (respectively, strictly positive). $\rho(A)$ is denoted as the spectral radius of matrix A. $\mathcal{E}[\cdot]$ means the mathematical expectation of $[\cdot]$. vec$[\cdot]$ is operation and consists of taking the columns of a given matrix from left to right and stack them one above the other. In symmetric block matrices, we use "$*$" as an ellipsis for the terms induced by symmetry, diag$\{\cdots\}$ for a block-diagonal matrix.

2. Problem formulation

Consider a class of discrete-time positive systems described as

$$\begin{aligned} x(k+1)&=Ax(k)\\ y(k)&=Cx(k) \end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the system state, $y(t)\in\mathbb{R}^p$ is the output. $A\in\mathbb{R}_+^{n\times n}$ and $C(\eta_t)\in\mathbb{R}_+^{p\times n}$ are known matrices.

In this paper, a new kind of positive observer with missing output is proposed as

$$\begin{aligned} \hat{x}(k+1)&=A\hat{x}(k)+\alpha(k)L(\hat{y}(k)-y(k))\\ \hat{y}(k)&=C\hat{x}(k) \end{aligned} \tag{2}$$

where $\hat{x}(t)$ is the estimate of $x(t)$, $\hat{y}(t)$ is the output, and $L\in\mathbb{R}_+^{n\times p}$ is the gain of the designed observer. The parameter $\alpha(k)$ represents the possible missing data process in output. It is assumed to be a discrete-time homogeneous Markov chain taking values in a finite set $\mathbb{S}=\{0,1\}$ with the following transition probability matrix:

$$\Pi=\begin{bmatrix}1-\alpha & \alpha\\ \beta & 1-\beta\end{bmatrix}$$

Here, parameters α and β are probabilities and defined as follows:

$$0<\alpha=\Pr\{\alpha(k+1)=1|\alpha(k)=0\}\le 1$$

and

$$0\le\beta=\Pr\{\alpha(k+1)=0|\alpha(k)=1\}\le 1$$

which are named as the recovery rate and failure rate respectively. If $\alpha+\beta=1$, $\alpha(k)$ will be reduced to the Bernoulli type missing data process, whose probability distributions are

$$\Pr\{\alpha(k)=1\}=\alpha, \Pr\{\alpha(k)=0\}=1-\alpha$$

Let $e(t)=\hat{x}(t)-x(t)$, if $e(t)\in\mathbb{R}_+^n$, we will have $\hat{x}(t)\in\mathbb{R}_+^n$, because of $x(t)\in\mathbb{R}_+^n$. Then, one gets

$$e(k+1)=[A+\alpha(k)LC]e(k) \tag{3}$$

Remark 1. It is said that positive observer (2) is actually a stochastic system, whose output experiences missing data and is described by a Markov chain. Compared with some existing references [17–20,28,29,31,32], the observer designed here is more general that the output is not necessary available online. Moreover, the probabilities describing such a missing data phenomenon will also be considered in the positive observer design. Though, $\alpha(k)$ is described by a Markov chain, it is said that system (2) or (3) is not a traditional Markovian jump system. In references [21–24,44–47], the system parameters switch synchronously according to a Markov process. Here, all the system parameters are deterministic, in which only the missing data in output is modeled into a Markov process.

In order to process our main results, some definitions for system (3) are introduced here.

Definition 1. System (3) is said to be positive, if and only if $e(t)\in\mathbb{R}_+^n$, for any $e(0)\in\mathbb{R}_+^n$ and $\forall\alpha(0)\in\mathbb{S}$.

Definition 2. The positive system in (3) is said to be asymptotically mean stable, if and only if the solution to system (3) satisfies

$$\lim_{k\to\infty}\mathcal{E}[e(k)]=0$$

with any initial condition $e(0)\in\mathbb{R}_+^n$ and $\forall\alpha(0)\in\mathbb{S}$.

Theorem 1. *System (3) is positive if and only if $A\in\mathbb{R}_+^{n\times n}$, $A+LC\in\mathbb{R}_+^{n\times n}$.*

Proof. Define stopping time instants $0=k_0^{i_0}<k_1^{i_1}<k_2^{i_0}<\cdots<\cdots$ belonging to $\mathbb{Z}^+$ and satisfying $\alpha(k_0^{i_0})=i_0$, $\alpha(k_1^{i_1})=i_1$, $\alpha(k_2^{i_0})=i_0$, $\cdots$, we have $\alpha(k)=\alpha(k_0^{i_0})=i_0$, $\forall k\in\mathbb{Z}^+[k_0^{i_0},k_1^{i_1})$, $\alpha(k)=\alpha(k_1^{i_1})=i_1$, $\forall k\in\mathbb{Z}^+[k_1^{i_1},k_2^{i_0}),\cdots$. Here, $i_0\in\mathbb{S}$ can be any value, but $i_1\in\mathbb{S}$ should be another different value. That means $i_0\in\mathbb{S}$, $i_1\in\mathbb{S}$, but $i_0\ne i_1$. Without loss of generality, it is assumed that $i_0=0$. Then, based on the definition of stopping time instants and let $e(0)\in\mathbb{R}_+^n$, we have

$$e(k+1)=Ae(k), \forall k\in\mathbb{Z}^+[k_0^{i_0},k_1^{i_1}) \tag{4}$$

It is very known that for system (4), $e(k)\in\mathbb{R}_+^n$, if and only if $A\in\mathbb{R}_+^n$. As for $\forall k\in\mathbb{Z}^+[k_1^{i_1},k_2^{i_0})$, by the similar method, one gets

$$e(k+1)=(A+LC)e(k), \forall k\in\mathbb{Z}^+[k_1^{i_1},k_2^{i_0}) \tag{5}$$

where $e(k_1^{i_1})=e(k_1^{i_1}-1)$. By system (4), it is seen that $e(k_1^{i_1}-1)\in\mathbb{R}_+^n$. Then, it is obtained that system (5) is positive with $e(k_1^{i_1})\in\mathbb{R}_+^n$ if and only if $A+LC\in\mathbb{R}_+^n$. By the step method, one has $e(k)\in\mathbb{R}_+^n$ on the intervals $\mathbb{Z}^+[k_2^{i_0},k_3^{i_1})$, $\mathbb{Z}^+[k_3^{i_1},k_4^{i_0})$, $\cdots$. Finally, we have $e(k)\in\mathbb{R}_+^n$ on $\mathbb{Z}^+$. This completes the proof.□

3. Main results

Theorem 2. *The following statements are equivalent.*

(i) *The positive system in (3) is asymptotically mean stable.*
(ii) *There exist strictly positive vectors (SPVs) $p_i\in\mathbb{R}_+^n$, $i\in\mathbb{S}$, such that*

$$A_i^T\tilde{p}_i-p_i\prec 0 \tag{6}$$

where

$$A_0=A, \tilde{p}_0=(1-\alpha)p_0+\alpha p_1$$
$$A_1=A+LC, \tilde{p}_1=\beta p_0+(1-\beta)p_1$$

(iii) *The following matrix* $\hat{A} \in \mathbb{R}_+^{2n \times 2n}$

$$\hat{A} = \begin{bmatrix} (1-\alpha)A_0 & \beta A_1 \\ \alpha A_0 & (1-\beta)A_1 \end{bmatrix} \quad (7)$$

is Schur.

(iv) *The following positive system*

$$x(k+1) = \hat{A}x(k) \quad (8)$$

is asymptotically stable. Equivalently, there is a positive define matrix $P \in \mathbb{R}^{2n \times 2n}$ *satisfying*

$$\hat{A}^T P \hat{A} - P < 0 \quad (9)$$

Proof. (i) ⇔ (iii) Define an indicator function

$$\mathbf{1}_{\{\alpha(k)=i\}}(\omega) = \begin{cases} 1 & \text{if} \alpha(k) = i \in \mathbb{S} \\ 0 & \text{otherwise} \end{cases}$$

Let $q(k) = \text{vec}[q_{i+1}(k)]$ with $q_{i+1}(k) = \mathcal{E}[e(k)\mathbf{1}_{\{\alpha(k)=i\}}(\omega)]$, it is concluded that

$$\mathcal{E}[e(k)] = \mathcal{E}[e(k)\mathbf{1}_{\{\alpha(k)=0\}}(\omega)] + \mathcal{E}[e(k)\mathbf{1}_{\{\alpha(k)=1\}}(\omega)] = q_1(k) + q_2(k) \quad (10)$$

where $q_1(k)$ and $q_2(k)$ are given as

$$q_1(k+1) = \mathcal{E}[A_0 e(k)\mathbf{1}_{\{\alpha(k+1)=0\}}\mathbf{1}_{\{\alpha(k)=0\}}] + \mathcal{E}[A_1 e(k)\mathbf{1}_{\{\alpha(k+1)=0\}}\mathbf{1}_{\{\alpha(k)=1\}}] = (1-\alpha)A_0 q_1(k) + \beta A_1 q_2(k)$$

$$q_2(k+1) = \mathcal{E}[A_0 e(k)\mathbf{1}_{\{\alpha(k+1)=1\}}\mathbf{1}_{\{\alpha(k)=0\}}] + \mathcal{E}[A_1 e(k)\mathbf{1}_{\{\alpha(k+1)=1\}}\mathbf{1}_{\{\alpha(k)=1\}}] = \alpha A_0 q_1(k) + (1-\beta)A_1 q_2(k)$$

Then, (10) is rewritten to be

$$q(k+1) = \hat{A}q(k) \quad (11)$$

Because of both systems (3) and (11) positive, it is claimed that the asymptotically mean stability of system (3) is equivalent to the asymptotic stability of system (11). As we know, system (11) being asymptotically stable is equivalent to $\rho(\hat{A}) = \rho(\hat{A}^T) < 1$ which is also referred to $\hat{A}$ being Schur stable.

(ii) ⇔ (iii) By reference [28], it is got that $\rho(\hat{A}) = \rho(\hat{A}^T) < 1$, if and only if there exists an SPV $p \in \mathbb{R}_+^{2n}$ satisfying

$$(\hat{A} - I_{2n})^T p \prec 0 \quad (12)$$

By letting $p = [p_1^T \quad p_2^T]^T$ and substituting $\hat{A}$ and p into (12), one will have (6).

(iii) ⇔ (iv) Based on the above analysis, it is obtained that the asymptotically mean stability of system (3) is equivalent to the asymptotic stability of system (11). Actually, systems (11) and (8) are same. As we know, deterministic system (11) is asymptotically stabile if and only if there exists a positive define matrix satisfying (9).

This completes the proof.□

Remark 2. It is seen that several equivalent conditions are presented with different forms. Here, the probabilities of missing data in output are considered in such stability conditions, which are also taken into account in the observer design. Though each of them can be used to check the stability conveniently, which one to be selected should depend on the concrete situations. Firstly, strict LMI condition (9) is very suitable to compute the observer parameter directly. Secondly, condition (6) is very convenient to be extended to some general cases that probabilities α and β are uncertain and unknown respectively. They will also make the design of the positive observer easily. Thirdly, because of considering the positivity of the studied systems, condition (6) is seemed to be obtained by exploiting a linear copositive stochastic Lyapunov function. Thus, its computational complexity will be smaller that ones without considering the positivity constraint.

Theorem 3. *Consider positive system (1), there exists positive observer (2) such that error system (3) is asymptotically mean stable if there exist* $P_{11} \in \mathbb{R}^{n \times n}$ *with* $P_{11} > 0$, $P_{12} \in \mathbb{R}^{n \times n}$ *with* $P_{12} = P_{12}^T$, $P_{22} \in \mathbb{R}^{n \times n}$ *with* $P_{22} > 0$, $G \in \mathbb{R}^{n \times n}$ *with* $G = (g_{st}) > 0$ *and* $Y \in \mathbb{R}_+^{n \times p}$, *satisfying*

$$g_{st} \le 0, \quad \forall s \ne t \in \{1, 2, \ldots, n\} \quad (13)$$

$$\begin{bmatrix} P & \Omega \\ * & 2\hat{G} - P \end{bmatrix} > 0 \quad (14)$$

where

$$P = \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix}, \hat{G} = \begin{bmatrix} G & 0 \\ * & G \end{bmatrix}, \Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}$$

$$\Omega_{11} = (1-\alpha)A^T G, \Omega_{12} = \alpha A^T G, \Omega_{21} = \beta A^T G + \beta C^T Y^T$$

$$\Omega_{22} = (1-\beta)A^T G + (1-\beta)C^T Y^T$$

Then, the gain of the designed observer is computed by

$$L = G^{-1} Y \quad (15)$$

Proof. Based on the result in [48], it is obtained that condition (9) is equivalent to

$$\begin{bmatrix} P & \hat{A}^T \hat{G}^T \\ * & \hat{G} + \hat{G}^T - P \end{bmatrix} > 0 \quad (16)$$

where $\hat{G}$ is any nonsingular matrix. By letting $\hat{G} = \text{diag}\{G, G\} > 0$, (16) will be rewritten to be

$$\begin{bmatrix} P & \hat{A}^T \hat{G}^T \\ * & 2\hat{G} - P \end{bmatrix} > 0 \quad (17)$$

where $G > 0$, and $\hat{A}^T \hat{G}^T$ can be computed as

$$\begin{bmatrix} (1-\alpha)A^T G & \alpha A^T G \\ \beta A^T G + \beta C^T L^T G & (1-\beta)A^T G + (1-\beta)C^T L^T G \end{bmatrix}$$

By the representation of (15), $\hat{A}^T \hat{G}^T$ is equivalent to Ω. From (14), it is known that $G > 0$. By condition (13), it is known that $-G$ is Hurwitz and $-G \in \mathbb{M}_n$. Then, based on the result in [49], one has $-G^{-1} \preceq 0$ which is equivalent to $G^{-1} \in \mathbb{R}_+^{n \times n}$. Since $Y \in \mathbb{R}_+^{n \times p}$, it is obtained $L = YG^{-1} \in \mathbb{R}_+^{n \times p}$. Thus, $A + LC \in \mathbb{R}_+^{n \times n}$. By Theorem 1, it is known that error system (3) is positive. This completes the proof.□

Theorem 4. *Consider positive system (1), there exists positive observer (2) such that error system (3) is asymptotically mean stable if there exist* $\hat{P} \in \mathbb{R}^{n \times n}$ *with* $\hat{P} = (\hat{p}_{st}) > 0$ *and* $Y \in \mathbb{R}_+^{n \times p}$, *satisfying*

$$\hat{p}_{st} \le 0, \quad \forall s \ne t \in \{1, 2, \ldots, n\} \quad (18)$$

$$\begin{bmatrix} -P & \overline{\Omega} \\ * & -P \end{bmatrix} < 0 \quad (19)$$

where

$$P = \begin{bmatrix} \hat{P} & 0 \\ * & \hat{P} \end{bmatrix}, \overline{\Omega} = \begin{bmatrix} \overline{\Omega}_{11} & \overline{\Omega}_{12} \\ \overline{\Omega}_{21} & \overline{\Omega}_{22} \end{bmatrix}$$

$$\overline{\Omega}_{11} = (1-\alpha)A^T \hat{P}, \overline{\Omega}_{12} = \alpha A^T \hat{P}$$

$$\overline{\Omega}_{21} = \beta A^T \hat{P} + \beta C^T Y^T$$

$$\overline{\Omega}_{22} = (1-\beta)A^T \hat{P} + (1-\beta)C^T Y^T$$

Then, the gain of the designed observer is computed by

$$L = \hat{P}^{-1} Y \tag{20}$$

Proof. By the Schur complement lemma, it is concluded that condition (9) is equivalent to

$$\begin{bmatrix} -P & \hat{A}^T \\ ** & -P^{-1} \end{bmatrix} < 0 \tag{21}$$

which is also equal to

$$\begin{bmatrix} -P & \hat{A}^T P \\ * & -P \end{bmatrix} < 0 \tag{22}$$

Let $P = \text{diag}\{\hat{P}, \hat{P}\} > 0$ and consider (20), it is known that $\hat{A}^T P$ is the same to $\overline{\Omega}$ in (19). The next proof is similar to the process of Theorem 3, which is omitted. This completes the proof.□

Remark 3. Here, LMI conditions for computing observer parameter L are given in Theorems 3 and 4, where the coupling between L and P has been decoupled in the former one. However, at the present forms, it is difficult to conclude which one is less conservative. So, it is said that both of them can be used to design positive observer and could be solved by using the Matlab toolbox directly.

Remark 4. Based on the developed model considering the probabilities of data missing, the relationship between such probabilities and existence conditions of positive observer are established successfully. On the one hand, by investigating references [27–32], it is seen that the output of the desired observer should be available online. However, this may be an ideal assumption in many practical applications. Different from these references, the proposed results have removed this assumption and could bear its data missing with some probabilities. Because of having less constrictions on the output, it is said that they will have wider application scope. On the other hand, instead of ignoring these probabilities totally, the presented results are less conservative since such probabilities are included. It is claimed that the larger the probability of the output is transmitted successfully, the less the conservatism of the obtained result is. Numerical examples will also demonstrate this phenomenon.

In Theorems 3 and 4, it is seen that the observer parameters are computed directly. When there are some general conditions acted on probabilities α and β, these forms will make the design of positive observer here very complicated. In order to deal with such general probabilities, another theorem will be proposed, where the observer parameter will be constructed.

Theorem 5. *Consider positive system (1), there exists positive observer (2) such that the resulting error system is asymptotically mean stable, if there are SPVs $p_i \in \mathbb{R}_+^n$, $\forall i \in \mathbb{S}$, and $\xi \in \mathbb{R}_+^p$, such that*

$$A^T \tilde{p}_0 - p_0 \prec 0 \tag{23}$$

$$A^T \tilde{p}_1 + C^T \xi - p_1 \prec 0 \tag{24}$$

Then, the corresponding gain of observer (2) is constructed as

$$L = \frac{v \xi^T}{v^T \tilde{p}_1} \tag{25}$$

where $v \in R_+^n$ is any given nonzero vector.

Proof. From condition (6), it is known that conditions (23) and

$$(A + LC)^T \tilde{p}_1 - p_1 \prec 0 \tag{26}$$

are necessary and sufficient conditions for the resulting error system asymptotically mean stable. By Theorem 1, it is known that $L \in \mathbb{R}_+^{n \times p}$ should be an SPV such that $A + LC \in \mathbb{R}_+^{n \times n}$. By the representation of L, it is seen that this assumption is obvious. Applying (26) to the original system, we have

$$A^T \tilde{p}_1 + C^T \frac{\xi v^T}{v^T \tilde{p}_1} - p_1 \prec 0 \tag{27}$$

which is equivalent to (26). This completes the proof.□

Remark 5. It is worth to point out that though probabilities α and β are included here, only β is used to construct observer parameter L. It means the current state estimate can be got from the designed observer if and only if the former output is available. Then, it is said that the representation (25) of parameter L is in accord with the facts coming from the problems and definitions related to observer (2).

From Theorem 5, it is seen that probabilities α and β play an important role in the positive observer design and should be given exactly. But in some applications, it is very hard or high cost to obtain them exactly, even they are totally unknown. Thus, it is natural and important to study such general cases. If there exist uncertainties in α and β, we will use their estimations which are described as

$$\alpha = \tilde{\alpha} + \Delta\tilde{\alpha}, \quad \tilde{\alpha} \in [0, 1] \tag{28}$$

$$\beta = \tilde{\beta} + \Delta\tilde{\beta}, \quad \tilde{\beta} \in [0, 1] \tag{29}$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are their estimates, and admissible uncertainties are $\Delta\tilde{\alpha} \in [-\epsilon, \epsilon]$ with $\epsilon \in [0, 1]$ and $\Delta\tilde{\beta} \in [-\delta, \delta]$ with $\delta \in [0, 1]$ respectively. Then, based on Theorem 5, we will have the following theorem.

Theorem 6. *Consider positive system (1), there exists positive observer (2) with conditions (28) and (29) such that the resulting error system is robustly asymptotically mean stable, if there are SPVs $p_i \in \mathbb{R}_+^n$, $\forall i \in \mathbb{S}$, $w \in \mathbb{R}_+^n$, $z \in \mathbb{R}_+^n$ and $\xi \in \mathbb{R}_+^p$, such that*

$$A^T p_0 - p_0 + (\tilde{\alpha} - \epsilon) A^T (p_1 - p_0) + 2\epsilon w \prec 0 \tag{30}$$

$$A^T (p_1 - p_0) - w \preceq 0 \tag{31}$$

$$A^T p_1 - p_1 + C^T \xi + (\tilde{\beta} - \delta)[A^T (p_0 - p_1) + C^T \xi (\kappa - 1)] + 2\delta z \prec 0 \tag{32}$$

$$A^T (p_0 - p_1) + (\kappa - 1) C^T \xi - z \preceq 0 \tag{33}$$

$$\kappa p_1 \preceq p_0 \tag{34}$$

where κ is a given nonnegative scalar. Then, the corresponding gain of observer (2) is constructed as

$$L = \frac{v \xi^T}{v^T p_1} \tag{35}$$

where $v \in R_+^n$ is any given nonzero vector.

Proof. Based on Theorem 2, it is obtained that system (3) is robustly asymptotically mean stable on all the admissible uncertainties, if the following conditions are satisfied.

$$A^T [(1 - \alpha) p_0 + \alpha p_1] - p_0 \prec 0 \tag{36}$$

$$(A + LC)^T [\beta p_0 + (1 - \beta) p_1] - p_1 \prec 0 \tag{37}$$

Substituting (28) and (29) into (36) and (37) respectively, we have

$$A^T p_0 - p_0 + \tilde{\alpha} A^T (p_1 - p_0) + \Delta\tilde{\alpha} A^T (p_1 - p_0) \prec 0$$
$$(A + LC)^T p_1 - p_1 + \tilde{\beta} (A + LC)^T (p_0 - p_1) + \Delta\tilde{\beta} (A + LC)^T (p_0 - p_1) \prec 0$$

which are equivalent to

$$A^T p_0 - p_0 + (\tilde{\alpha} - \epsilon) A^T (p_1 - p_0) + (\Delta\tilde{\alpha} + \epsilon)[A^T (p_1 - p_0) - w] + (\Delta\tilde{\alpha} + \epsilon) w \prec 0 \tag{38}$$

$$(A + LC)^T p_1 - p_1 + (\tilde{\beta} - \delta)(A + LC)^T (p_0 - p_1)$$

$$+\Delta\tilde{\beta}+\delta)[(A+LC)^T(p_0-p_1)-z]+(\Delta\tilde{\beta}+\delta)z\prec 0 \quad (39)$$

where $w\in\mathbb{R}^n_+$, $z\in\mathbb{R}^n_+$. From definition (28), it is claimed that (38) is implied by (30) and (31). As for (39), based on definition (29), we have it ensured by

$$(A+LC)^Tp_1-p_1+(\tilde{\beta}-\delta)(A+LC)^T(p_0-p_1)+2\delta z\prec 0 \quad (40)$$

$$(A+LC)^T(p_0-p_1)-z\prec 0 \quad (41)$$

Substituting (35) into (40), one gets

$$A^Tp_1-p_1+C^T\frac{\xi v^T}{v^Tp_1}p_1+(\tilde{\beta}-\delta)(A^T+C^T\frac{\xi v^T}{v^Tp_1})(p_0-p_1)+2\delta z\prec 0 \quad (42)$$

Taking into account (34), we have (42) guaranteed by

$$A^Tp_1-p_1+C^T\xi+(\tilde{\beta}-\delta)A^T(p_0-p_1)+C^T\frac{\xi v^T}{v^Tp_1}(\kappa p_1-p_1)+2\delta z\prec 0$$

which is equivalent to (32). Similarly, (41) can be obtained by

$$A^T(p_0-p_1)+C^T\frac{\xi v^T}{v^Tp_1}(\kappa p_1-p_1)-z\prec 0 \quad (43)$$

which is actual (33). This completes the proof.□

When probabilities α and β satisfy another general case that both of them are unknown, how to design the proper observer gain is also an interesting question. Similar to Theorem 6, we have the following result.

Theorem 7. *Consider positive system (1), there exists positive observer (2) with unknown α and β such that the resulting error system is asymptotically mean stable, if there are SPVs $p_i\in\mathbb{R}^n_+$, $\forall i\in\mathbb{S}$, $w\in\mathbb{R}^n_+$, $z\in\mathbb{R}^n_+$ and $\xi\in\mathbb{R}^p_+$, satisfying condition (34) and*

$$A^Tp_0-p_0+w\prec 0 \quad (44)$$

$$A^T(p_1-p_0)-w\preceq 0 \quad (45)$$

$$A^Tp_1-p_1+C^T\xi+z\prec 0 \quad (46)$$

$$A^T(p_0-p_1)+(\kappa-1)C^T\xi-z\preceq 0 \quad (47)$$

Then, the corresponding gain of observer (2) with unknown α and β can also be constructed by (35).

Proof. Based on Theorem 2, it is obtained that system (3) is asymptotically mean stable, if the following conditions

$$A^Tp_0-p_0+\alpha A^T(p_1-p_0)\prec 0$$
$$(A+LC)^Tp_1-p_1+\beta(A+LC)^T(p_0-p_1)\prec 0$$

hold. Similar to the methods dealing with (36) and (37), we have both of them equal to

$$A^Tp_0-p_0+\alpha[A^T(p_1-p_0)-w]+\alpha w\prec 0 \quad (48)$$

$$(A+LC)^Tp_1-p_1+\beta[(A+LC)^T(p_0-p_1)-z]+\beta z\prec 0 \quad (49)$$

where $w\in\mathbb{R}^n_+$, $z\in\mathbb{R}^n_+$. Considering $\alpha\in[0,1]$ and $\beta\in[0,1]$, it is known that (48) is guaranteed by (48) and (45), while (49) is implied by

$$(A+LC)^Tp_1-p_1+z\prec 0 \quad (50)$$

$$(A+LC)^T(p_0-p_1)-z\preceq 0 \quad (51)$$

Substituting (35) into (50) and (51) respectively, we have

$$A^Tp_1-p_1+C^T\frac{\xi v^T}{v^Tp_1}p_1+z\prec 0 \quad (52)$$

$$A^T(p_0-p_1)+C^T\frac{\xi v^T}{v^Tp_1}(p_0-p_1)-z\prec 0 \quad (53)$$

It is obvious that (52) is actual (46), while (53) can be obtained by (47) if condition (34) is applied on (53). This completes the proof.□

Remark 6. It is worth mentioning that there are some possible approaches to further reduce the conservatism. Firstly, since the process of data missing is modeled by a Markov chain and the linear co-positive stochastic Lyapunov function is mode-dependent in terms of depending on the Markov process, it will be less conservatism if the desired observer is also mode-dependent. In this case, the common parameter L in (2) will be replaced by mode-dependent parameter $L(\alpha(k))$. But, how to achieve this aim should be considered carefully. For example, some additional problems will emerge, since the positivity should also be taken into account simultaneously. Secondly, it is thought that the more accurate the probabilities are, the less conservatism the obtained results will be. Thirdly, as for the cases of the probabilities satisfying the above general conditions, it is seen that some positive parameter and nonzero vector are given beforehand, and different values will lead to differences. But, what is the relationship between the conservatism of the results and the given constants should be revisited and will be our further work.

4. Numerical examples

Example 1. Consider a discrete-time positive system described by

$$x(k+1)=\begin{bmatrix}0.1595 & 0.1890 & 0.2713\\ 0.5091 & 0 & 0\\ 0 & 0.6740 & \zeta\end{bmatrix}x(k)$$
$$y(k)=[1\ \ 0\ \ 0]x(k)$$

where $\zeta\geq 0$ is a given scalar. For this example, the transition probability matrix of parameter $\alpha(k)$ is first assumed to be

$$\Pi=\begin{bmatrix}0.6 & 0.4\\ 0.6 & 0.4\end{bmatrix}$$

where $\alpha=0.4$, $\beta=0.6$. Without loss of generality, ζ is first assumed to be $\zeta=0$. By Theorem 3, we have matrices G and Y computed by solving LMIs (13) and (14) and given as

$$G=\begin{bmatrix}27.2876 & -7.2312 & -7.0515\\ -7.2312 & 26.3090 & -6.3669\\ -7.0515 & -6.3669 & 25.1749\end{bmatrix}, Y=\begin{bmatrix}11.2202\\ 9.3918\\ 12.0719\end{bmatrix}$$

which are positive-definitive matrix and strictly positive vector respectively. Then, the parameter of the desired observer can be computed by (15) and is given as

$$L=\begin{bmatrix}0.8693\\ 0.8211\\ 0.9307\end{bmatrix}$$

Under the initial condition $e_0=[2\ \ 5\ \ 8]^T$, we have the response of error $e(k)$ shown in Fig. 1. Moreover, the simulation of $\alpha(k)$ taking values in $\mathbb{S}=\{0,1\}$ is demonstrated in Fig. 2, where $\alpha(k)=0$ denotes the data of output missing. By Theorem 4, one can get the gain of L as

$$L=\begin{bmatrix}0.4135\\ 0.4458\\ 0.4877\end{bmatrix}$$

where the needed parameters of matrix $\hat{P}$ and SPV Y are got by solving LMIs (18) and (19) and given as

$$\hat{P}=\begin{bmatrix}161.0184 & -35.7056 & -36.4605\\ -35.7056 & 143.4002 & -35.7465\\ -36.4605 & -35.7465 & 124.9898\end{bmatrix}, Y=\begin{bmatrix}32.8858\\31.7229\\29.9460\end{bmatrix}$$

It is obvious that $\hat{P}$ is a positive-definitive matrix and Y is an SPV. Based on the proposed results in this paper, it is known that the parameter L can also be constructed by Theorem 5. Without loss of generality, if the corresponding nonzero vector v used in (25) is given as

$$v=\begin{bmatrix}0.2\\1\\0.5\end{bmatrix}$$

we could have the corresponding gain of observer (2) constructed as

$$L=\begin{bmatrix}0.0291\\0.1454\\0.0727\end{bmatrix}$$

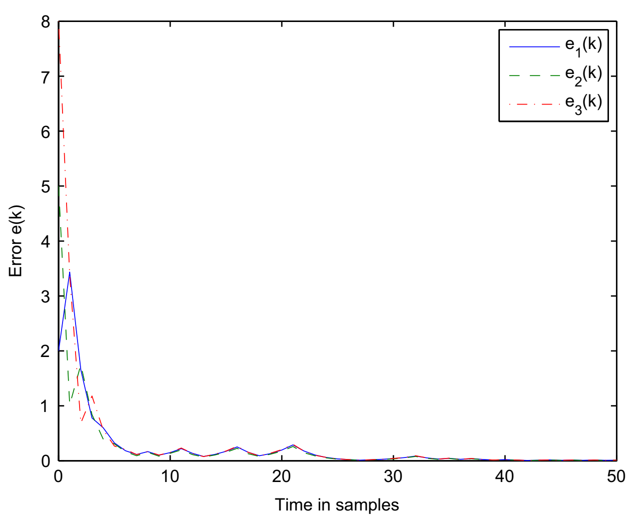


Fig. 1. The response of error $e(k)$.

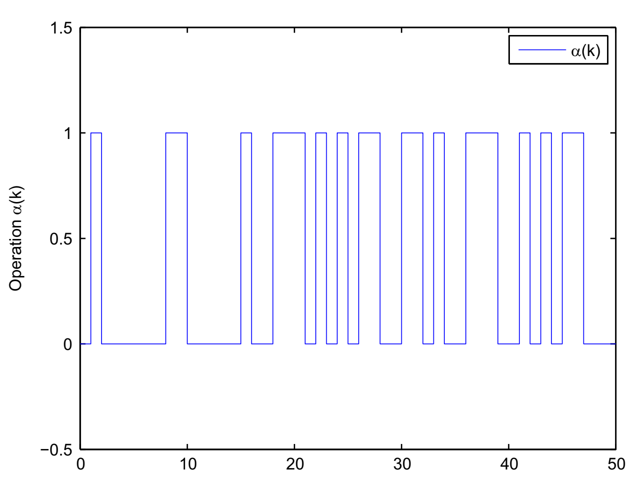


Fig. 2. The simulation of operation mode $\alpha(k)$.

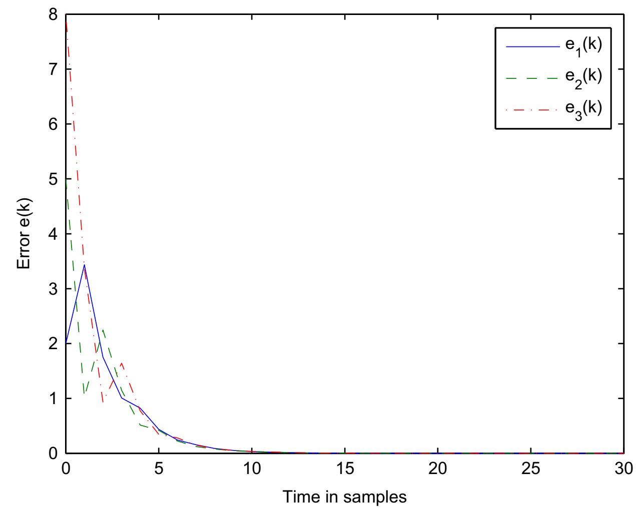


Fig. 3. The simulation of error $e(k)$.

Here, the corresponding SPVs p_0 and p_1 obtained by solving conditions (23) and (24) are given as

$$p_0=\begin{bmatrix}29.7937\\30.1105\\22.4569\end{bmatrix}, p_1=\begin{bmatrix}34.9264\\31.7510\\24.4722\end{bmatrix}$$

and $\xi=7.0918$. Under the same initial condition, we have the simulation of the resulting error system shown in Fig. 3. When probabilities α and β are uncertain and described as $\tilde{\alpha}=0.4$ with $\epsilon=0.4\tilde{\alpha}$ and $\tilde{\beta}=0.6$ with $\delta=0.4\tilde{\beta}$, under the same nonzero vector v and by solving conditions (30)–(34) with $\kappa=1.4$, we get SPVs p_1 and ξ as follows:

$$p_1=\begin{bmatrix}71.4437\\61.8469\\34.8618\end{bmatrix}, \xi=3.6928$$

Then, by (35), we have L computed as

$$L=\begin{bmatrix}0.0079\\0.0395\\0.0197\end{bmatrix}$$

If such probabilities are unknown, under the same conditions, by solving such conditions in Theorem 7, all the corresponding SPVs could be got and given as follows:

$$p_1=\begin{bmatrix}47.6262\\44.0927\\25.0128\end{bmatrix}, \xi=1.0298, L=\begin{bmatrix}0.0031\\0.0156\\0.0078\end{bmatrix}$$

Based on such simulations, it is seen that the proposed results are useful to deal with the proposed problems in this paper. Finally, when parameter ζ is taken as $\zeta=1$ and such probabilities are known exactly, it is concluded that there is no solution to L by Theorems 4 Theorems 5, while by Theorem 3 we have

$$L=\begin{bmatrix}0.6618\\0.5658\\0.8589\end{bmatrix}$$

For example, it is claimed that Theorem 3 is less conservative.

Example 2. Consider a discrete-time positive system described by

$$e(k+1)=(A+LC)e(k) \tag{54}$$

Table 1
Admissibly maximum ζ for different pair (α,β).

(α,β)	$\alpha=0$	$\alpha=0.1$	$\alpha=0.2$	$\alpha=0.3$	$\alpha=0.4$	$\alpha=0.5$
$\beta=0$	3.505	3.505	3.505	3.505	3.505	3.505
$\beta=0.1$	4.209	4.51	5.13	6.492	11.15	∞
$\beta=0.2$	5.772	6.506	8.139	14.09	∞	∞
$\beta=0.3$	21.75	∞	∞	∞	∞	∞
$\beta=0.4$	∞	∞	∞	∞	∞	∞
$\beta=0.5$	∞	∞	∞	∞	∞	∞

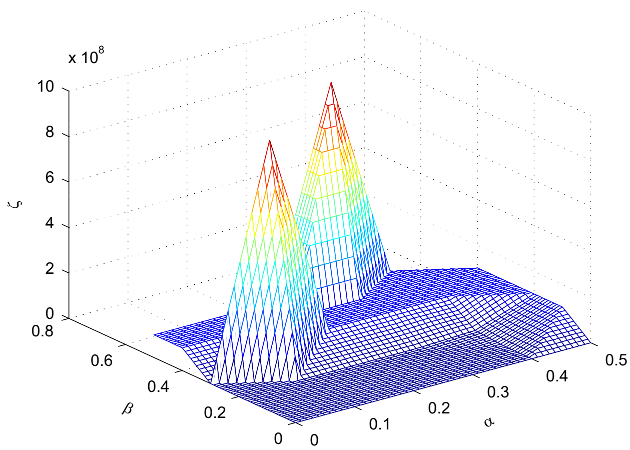


Fig. 4. The simulation of maximum ζ varying with (α,β).

where

$$A=\begin{bmatrix}2 & 3\\ 0 & \zeta\end{bmatrix}, C=[1 \quad 0], L=[0.0876 \quad 0.0406]$$

where $\zeta \geq 0$ is a given scalar. For this example, by the existing results on stability, it is known that positive (54) will be asymptotically stable if and only if $0 \leq \zeta \leq 3.505$. By the methods proposed in this paper, the probability of missing data in output is considered, and the corresponding model is described by

$$e(k+1)=(A+\alpha(k)LC)e(k) \tag{55}$$

where the transition probability matrix of parameter $\alpha(k)$ is assumed to be

$$\Pi=\begin{bmatrix}1-\alpha & \alpha\\ \beta & 1-\beta\end{bmatrix}$$

Without loss of generality, for this example, probabilities α and β take values in $[0, 0.5]$. By Theorem 2, we have the maximum value of ζ listed in Table 1. Here, "∞" means the admissibly maximum value of ζ under some pairs (α,β), which is very large compared with other finite values. From this example, it is claimed that because of such probabilities considered, our results are less conservative than ones obtained by the traditional methods. In addition, the correlation between maximum ζ and pair (α,β) is also demonstrated in Fig. 4. Based on such simulations, it is said that our results are effective and have some superiorities in terms of less conservatism.

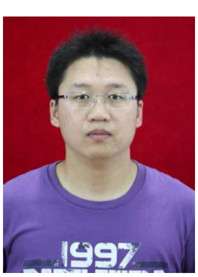
5. Conclusions

In this paper, we have studied the positive observer design for a kind of discrete-time positive system with missing data in output. A necessary and sufficient condition for the positivity has been addressed. Several kinds of equivalent conditions for checking the asymptotically mean stability have been proposed to be related to some probabilities, which can be solved easily. Based on the presented results, several sufficient conditions for the existence of the designed observer are given with solvable forms. Finally, some general cases that such probabilities are uncertain and unknown have been considered too. Future research could be the extensions of the proposed results to other systems such as time delay systems, singular systems, and the output control or filtering problems of discrete-time positive systems with missing data in output.

References

[1] L. Farina, S. Rinaldi, Positive Linear Systems: Theory and Applications, Wiley, New York, 2000.
[2] B. Cantó, C. Coll, E. Sánchez, Positive solutions of a discrete-time descriptor system, Int. J. Syst. Sci. 39 (1) (2008) 81–88.
[3] E. Klipp, R. Herwig, A. Kowald, C. Wierlig, H. Lehrach, Systems biology in practice: concepts, Implementation and Application, Wiley, Berlin, 2005.
[4] F. Wirth, R. Shorten, D. Leith, A positive system model of tcp-like congestion control: asymptotic results, IEEE/ACM Trans. Netw. 14 (3) (2006) 616–629.
[5] L. Benvenuti, L. Farina, A tutorial on the positive realization problem, IEEE Trans. Autom. Control 49 (5) (2004) 651–664.
[6] C. Commault, M. Alamir, On the reachability in any fixed time for positive continuous-time linear systems, Syst. Control Lett. 56 (4) (2007) 272–276.
[7] M.E. Valcher, On the reachability properties of continuous-time positive systems, in: Proceeding 16th Mediterranean Conference on Control Automation, Ajaccio, France, June 25–27, 2008, pp. 990–995.
[8] X. Liu, W. Yu, L. Wang, Stability analysis of continuous-time positive systems with time-varying delays, IEEE Trans. Autom. Control 55 (4) (2010) 1024–1028.
[9] E. Fornasini, M.E. Valcher, Stability and stabilizability criteria for discrete-time positive switched systems, IEEE Trans. Autom. Control 57 (5) (2012) 1208–1221.
[10] Y.M. Zhang, Q.L. Zhang, T. Tanaka, X.G. Yan, Positivity of continuous-time descriptor systems with time delays, IEEE Trans. Autom. Control 59 (11) (2014) 3093–3097.
[11] P. Bolzern, P. Colaneri, G.D. Nicolao, Stochastic stability of positive Markov jump linear systems, Automatica 50 (4) (2014) 1181–1187.
[12] X. Liu, L. Wang, W. Yu, S. Zhong, Constrained control of positive discrete-time systems with delays, IEEE Trans. Circuits Syst. II: Express Br. 55 (2) (2008) 193–197.
[13] J.E. Feng, J. Lam, P. Li, Z. Shu, Decay rate constrained stabilization of positive systems using state output feedback, Int. J. Robust Nonlinear Control 21 (1) (2011) 44–54.
[14] Y. Tong, C. Wang, L. Zhang, Stabilisation of discrete-time switched positive linear systems via time- and state-dependent switching laws, IET Control Theory Appl. 6 (11) (2012) 1603–1609.
[15] J.F. Zhang, Z.Z. Han, F.B. Zhu, J. Huang, Feedback control for switched positive linear systems, IET Control Theory Appl. 7 (3) (2014) 464–469.
[16] S.Q. Zhu, Q.L. Han, C.H. Zhang, l_1-gain performance analysis and positive filter design for positive discrete-time Markov jump linear systems: a linear programming approach, Automatica 50 (8) (2014) 2098–2107.
[17] A.M. Pertew, H.J. Marquez, Q. Zhao, H_∞ observer design for Lipschitz nonlinear systems, IEEE Trans. Autom. Control 51 (7) (2006) 1211–1216.
[18] S. Sundaram, C.N. Hadjicostis, Delayed observers for linear systems with unknown inputs, IEEE Trans. Autom. Control 52 (2) (2007) 334–339.
[19] H.S. Wu, Adaptive robust state observers for a class of uncertain nonlinear dynamical systems with delayed state perturbation, IEEE Trans. Autom. Control 54 (6) (2009) 1407–1412.
[20] L. Xie, P. Khargonekar, Lyapunov-based adaptive state estimation for a class of nonlinear stochastic systems, Automatica 48 (7) (2012) 1423–1431.
[21] J. Raouf and E.K. Boukas, Observer-based controller design for linear singular systems with Markovian switching, in: Proceeding 2004 Conference on Decision and Control, Atlantis, Paradise Island, Bahamas, December 14–17, 2004, pp. 3619–3624.
[22] X.M. Zhang, G.P. Lu, Y.F. Zheng, Observer design for descriptor Markovian jumping systems with nonlinear perturbations, Circuits Syst. Signal Process. 27 (1) (2008) 95–112.
[23] H.L. Dong, Z.D. Wang, H.J. Gao, Observer-based H_∞ control for systems with repeated scalar nonlinearities and multiple packet losses, Int. J. Robust Nonlinear Control 21 (12) (2010) 1363–1378.
[24] G.L. Wang, Q.L. Zhang, Adaptive state estimation for stochastic delay systems with state-dependent Markovian switching, IET Control Theory Appl. 6 (6) (2012) 822–828.
[25] H.L. Dong, Z.D. Wang, H.J. Gao, Distributed H_∞ filtering for a class of Markovian jump nonlinear time-delay systems over lossy sensor networks, IEEE Trans. Ind. Electron. 60 (10) (2013) 4665–4672.
[26] H.Y. Li, H.J. Gao, P. Shi, X.D. Zhao, Fault-tolerant control of Markovian jump stochastic systems via the augmented sliding mode observer approach, Automatica 50 (7) (2014) 1825–1834.
[27] J. Back and A. Astolfi, Existence conditions and a constructive design of positive linear observers for positive linear systems, in: Proceeding 45th IEEE Conference on Decision and Control, San Diego, USA, December 13–15, 2006, pp. 4734–4739.

[28] M.A. Rami and F. Tadeo, Positive observation problem for linear discrete positive systems, in: Proceeding 45th IEEE Conference on Decision and Control, San Diego, USA, December 13–15, 2006, pp. 4729–4733.
[29] H.M. Hardin, J.H.V. Schuppen, Observers for linear positive systems, Linear Algebr. Appl. 425 (2–3) (2007) 571–607.
[30] Z. Shu, J. Lam, H.J. Gao, B.Z. Du, Positive observer and dynamic output-feedback controllers for interval positive linear systems, IEEE Trans. Circuits Syst.-I: Regul. Pap. 55 (10) (2008) 3209–3222.
[31] B. Brain, J. Wang and Z. H. Qu, Nonlinear positive observer design for positive dynamical systems, in: 2010 American Control Conference, Baltimore, USA, June 30–July 02, 2010, pp. 6231–6237.
[32] M.A. Rami, F. Tadeo, U. Helmke, Positive observers for linear positive systems, and their implications, Int. J. Control 84 (4) (2011) 716–725.
[33] S.J. Qin, Survey on data-driven industrial process monitoring and diagnosis, Annu. Rev. Control 36 (2) (2012) 220–234.
[34] G. Battistelli, A. Benavoli, L. Chisci, Data-driven communication for state estimation with sensor networks, Automatica 48 (5) (2012) 926–935.
[35] Y.Q. Xia, W. Xie, B. Liu, X.Y. Wang, Data-driven predictive control for networked control systems, Inf. Sci. 235 (2013) 45–54.
[36] H.Y. Li, X.J. Jing, H.K. Lam, P. Shi, Fuzzy sampled-data control for uncertain vehicle suspension systems, IEEE Trans. Cybern. 44 (7) (2014) 1111–1126.
[37] S. Yin, S.X. Ding, X.C. Xie, H. Luo, A review on basic data-driven approaches for industrial process monitoring, IEEE Trans. Ind. Electron. 61 (11) (2014) 6418–6428.
[38] S. Yin, X.W. Li, H.J. Gao, O. Kaynak, Data-based techniques focused on modern industry: an overview, IEEE Trans. Ind. Electron. 62 (1) (2015) 657–667.
[39] L.Q. Zhang, Y. Shi, T.W. Chen, B. Huang, A new method for stabilization of networked control sysetms with random delays, IEEE Trans. Autom. Control 50 (8) (2005) 1177–1181.
[40] D. Yue, E.G. Tian, Y.J. Zhang, C. Peng, Delay-distribution-dependent robust stability of uncertain systems with time-varying delays, Int. J. Robust Nonlinear Control 19 (4) (2008) 377–393.
[41] D. Yue, E.G. Tian, Z.D. Wang, J. Lam, Stabilization of systems with probabilistic interval input delays and its applications to networked control systems, IEEE Trans. Syst. Man Cybern. Part A: Syst. Hum. 39 (4) (2009) 939–945.
[42] D. Huang, S.K. Nguang, State feedback control of uncertain networked control systems with random delays, IEEE Trans. Autom. Control 53 (3) (2008) 829–834.
[43] M. Liu, D.W.C. Ho, Y.G. Niu, Stabilization of Markovian jump linear system over networks with random communication delay, Automatica 45 (2) (2009) 416–421.
[44] T. Shi, H.Y. Su, Jian. Chu, Robust H_∞ output feedback control for Markovian jump systems with actuator saturation, Optim. Control Appl. Methods 33 (6) (2012) 676–695.
[45] H.Y. Li, X.J. Jing, H.R. Karimi, Output-feedback based H_∞ control for active suspension systems with control delay, IEEE Trans. Ind. Electron. 61 (1) (2014) 436–446.
[46] Z.G. Wu, P. Shi, H.Y. Su, Jian. Chu, Stochastic synchronization of Markovian jump neural networks with time-varying delay using sampled-data, IEEE Trans. Cybern. 43 (6) (2013) 1796–1806.
[47] Z.G. Wu, P. Shi, H.Y. Su, Jian. Chu, Asynchronous l_2–l_∞ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities, Automatica 50 (1) (2014) 180–186.
[48] M.C. de Oliveira, J. Bernussou, J.C. Geromel, A new discrete-time robust stability condition, Syst. Control Lett. 37 (4) (1999) 261–265.
[49] O. Mason, R. Shorten, On linear copositive Lyapunov functions and the stability of switched positive linear systems, IEEE Trans.Autom. Control 52 (7) (2007) 1346–1349.



Guoliang Wang received the B.Sc., M.Sc. and Ph.D. degrees in control theory and engineering from the Northeastern University of Shenyang, China, in 2004, 2007 and 2010, respectively. He is presently an associate professor in the school of information and control engineering, Liaoning Shihua University, Fushun, Liaoning, China. He has authored/co-authored over 40 publications. His research interests include Markovian jump systems, singular systems, stochastic control and filtering.

Boyu Li is graduate candidate at Liaoning Shihua University. His research interest includes Markovian jump systems, singular systems and robust control systems.

Qingling Zhang received the B.S. and M.S. degrees from the Mathematics Department and the Ph.D. degree from the Automatic Control Department of Northeastern University, Shenyang, China, in 1982, 1986, and 1995, respectively. Since 1997, he has been a Professor with Northeastern University. He is also a member of the University Teaching Advisory Committee of National Ministry of Education. He has published six books and more than 230 papers about control theory and applications. Dr. Zhang received 14 prizes from central and local governments for his research. He has also received the Golden Scholarship from Australia in 2000. His current research interests cover singular systems, networked control systems, and robust control.

Chunyu Yang received the B.Sc., M.Sc. and Ph.D. degrees in control theory and engineering from the Northeastern University of Shenyang, China, in 2002, 2004 and 2009, respectively. He is presently an associate professor in the school of information and electrical engineering, China University of Mining and Technology, Xuzhou, China. His research interests include descriptor systems, singular perturbed systems, and electrical systems.