Дата / _____ /

$$y=\int\frac{x^3-6x^2+11x-6}{\sqrt{x^2+4x+2}}dx=(Ax^2+Bx+C)\cdot\sqrt{x^2+4x+2}+\lambda\cdot\int\frac{dx}{\sqrt{x^2+4x+2}}$$

$$\frac{x^3-6x^2+11x-6}{\sqrt{x^2+4x+2}}=(2Ax+B)\cdot\sqrt{\dots}+(Ax^2+Bx+C)\cdot\frac{2x+4}{2\sqrt{x^2+4x+2}}+\lambda\cdot\frac{1}{\sqrt{x^2+4x+2}}\quad\Big|\times\sqrt{\dots}$$

$$x^3-6x^2+11x-6=(2Ax+B)(x^2+4x+2)+(Ax^2+Bx+C)\cdot(x+2)+\lambda$$

x^3	$2A+A=1$	$A=\frac{1}{3}$	
x^2	$8A+B+2A+B=-6$	$\frac{10}{3}+2B=-6$	$2B=\frac{-28}{3}$
x	$4A+4B+2B+C=11$	$\frac{4}{3}-28+C=11$	$B=-\frac{14}{3}$
1	$2B+2C+\lambda=-6$	$C=39-\frac{4}{3}=\frac{39\cdot3-4}{3}$	

$$y=\left(\frac{1}{3}x^2+\frac{-14}{3}x+\frac{39\cdot3-4}{3}\right)\sqrt{x^2+4x+2}+\underbrace{?}_{\lambda=\dots}\int\frac{dx}{\sqrt{x^2+4x+2}}$$

$$\int\frac{d(x+2)}{\sqrt{(x+2)^2-(\sqrt{2})^2}}=\ln\left|x+2+\sqrt{\underbrace{x^2+4x+2}_{x^2+2\cdot x\cdot 2+4-2}}\right|+C$$

$$\int\frac{1\cdot dx}{(x-1)^2\sqrt{1+2x-x^2}}=$$

$x>1$ $\quad x-1=\frac{1}{t};\ x=\frac{1}{t}+1$

$t=\frac{1}{x-1}$ $\quad dx=-\frac{1}{t^2}dt$

$1+2x-x^2=1+2\left(\frac{1}{t}+1\right)-\left(\frac{1}{t}+1\right)^2=$

$=1+\frac{2}{t}+2-\frac{1}{t^2}-\frac{2}{t}+1=$

$=-\frac{1}{t^2}+4=\frac{-1+4t^2}{t^2}$

$$=\int\frac{-t^2\,dt}{t^2\cdot\frac{\sqrt{-1+4t^2}}{t}}=\quad dx$$

$$=-\int\frac{t\,dt}{\sqrt{4t^2-1}}=\quad \frac{1}{8}d(4t^2-1)$$

$$=-\frac{1}{8}\int\frac{d(4t^2-1)}{\sqrt{4t^2-1}}=\left|\begin{array}{l}u=4t^2-1\\ \int\frac{du}{\sqrt{u}}=\int u^{-\frac{1}{2}}du=\frac{u^{-\frac{1}{2}+1}}{\frac{1}{2}+1}+C\end{array}\right|=-\frac{1}{8}\frac{\sqrt{4t^2-1}}{1/2}+C$$

$$=-\frac{1}{4}\sqrt{4\left(\frac{1}{x-1}\right)^2-1}+C$$

BUROMAX
Максимум якісної канцелярії

$$\int \frac{x\,dx}{\sqrt{1+\sqrt[3]{x^2}}} = \int x\left(1+x^{\frac{2}{3}}\right)^{-\frac{1}{2}} dx = \boxed{\int x^m (a+bx^n)^p dx}$$

$m=1,\ n=\frac{2}{3},\ p=-\frac{1}{2}$ | Вип I $p \notin \mathbb{Z}$ | Вип II $\frac{m+1}{n} \notin \mathbb{Z}$

$a=1,\ b=1$ | $\frac{1+1}{2/3} = 2 : \frac{2}{3} = 2 \cdot \frac{3}{2} = 3 \in \mathbb{Z}$

$t = \sqrt[\nu]{a+bx^n}$, ν – знам p. $\nu = 2 \Rightarrow t = \sqrt{1+x^{2/3}}$

$t^2 = 1 + x^{2/3}$ $\quad t^2 - 1 = x^{2/3}$ $\quad x = (t^2-1)^{3/2}$

$dx = \frac{3}{2} \cdot (t^2-1)^{1/2} \cdot 2t\,dt$

$$= \int \frac{(t^2-1)^{3/2} \cdot \frac{3}{\cancel{2}}(t^2-1)^{1/2} \cdot \cancel{2}\cancel{t}\,dt}{\cancel{t}} = 3\int (t^2-1)^2 dt =$$

$$= 3\int (t^4 - 2t^2 + 1)\,dt = 3\left(\frac{t^5}{5} - 2\cdot\frac{t^3}{3} + t\right) + C =$$

$$= 3\left(\frac{1}{5}\left(\sqrt{1+x^{2/3}}\right)^5 - \frac{2}{3}\left(\sqrt{1+x^{2/3}}\right)^3 + \sqrt{1+x^{2/3}}\right) + C$$

$$\int sh^4 x\,dx = \int \frac{1}{4}(ch^2 2x - 2ch 2x + 1)\,dx$$

$$= \frac{1}{4}\left(-\cancel{2}\cdot\frac{1}{\cancel{2}} sh 2x + x + \frac{1}{2}\int (ch 4x + 1)\,dx\right)$$

$$= \frac{1}{4}\left(-sh 2x + x + \frac{1}{2}\left[\frac{1}{4} sh 4x + x\right]\right) + C$$

$\int sh^\mu x\, ch^\nu x\,dx$

$sh^2 x = \frac{1}{2}(ch 2x - 1)$

$ch^2 x = \frac{1}{2}(ch 2x + 1)$

$$\int sh^3 x\,dx = \int sh^2 x \cdot \underbrace{sh x\,dx}_{= d(ch x)} = \int (ch^2 x - 1)\,d(ch x)$$

$ch^2 x - sh^2 x = 1$ $\quad t = ch x$ $\quad = \int (t^2 - 1)\,dt = \ldots$

$$\int \frac{dx}{sh x + 2 ch x} = \left| \begin{array}{l} t = e^x \\ x = \ln t \end{array} \quad dx = \frac{1}{t}dt \quad sh x = \frac{t^2-1}{t} \quad ch x = \frac{t^2+1}{t} \right| =$$

$$= \int \frac{\frac{1}{t}dt}{\frac{t^2-1}{t} + 2\frac{t^2+1}{t}} = \int \frac{dt}{t^2 - 1 + 2(t^2+1)} = \int \frac{dt}{3t^2+1} =$$

$y^2 = 3t^2$ $\quad y = t\sqrt{3}$

$a^2 = 1$ $\quad a = 1$

$$\int \frac{dy}{y^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{y}{a} + C$$

$$= \frac{1}{\sqrt{3}} \cdot \frac{1}{1} \operatorname{arctg} \frac{t\sqrt{3}}{1} + C = \frac{1}{\sqrt{3}} \operatorname{arctg} \sqrt{3}\, e^x$$

$$\int\left(1-\frac{2}{x}\right)^2 e^x\,dx=\int\left(1-\frac{4}{x}+\frac{4}{x^2}\right)e^x\,dx=$$

$$li\,x=\int\frac{dx}{\ln x}$$

$$\int\frac{e^{\alpha x}}{x}dx=li(e^{\alpha x})+C$$

$$=x-4\int\frac{e^x}{x}dx+4\int\frac{e^x}{x^2}dx=A$$

$$=x-4li(e^x)+4\cdot A=\left|\begin{array}{ll}u=e^x & du=e^x dx\\ dv=\frac{dx}{x^2} & v=\frac{x^{-2+1}}{-2+1}=-\frac{1}{x}\end{array}\right|=$$

$$=x-4li(e^x)+4\left(-\frac{1}{x}e^x+\int\frac{e^x}{x}dx\right)=$$

$$-\frac{3}{4}-\frac{1}{2}=\frac{-3-2}{4}$$

$$=x-\cancel{4li(e^x)}-\frac{4}{x}e^x+\cancel{4li(e^x)}+C=x-\frac{4e^x}{x}+C$$

$$\int\frac{dx}{\cos x\cdot\sqrt[3]{\sin^2 x}}=\int\frac{\cos x\,dx}{\cos^2 x\,(\sin x)^{2/3}}=\qquad \cos x\,dx=d(\sin x)$$

$$\int\sin^\mu x\cos^\nu x\,dx$$

$$\mu=-\frac{2}{3},\ \nu=-1$$

$$=\int\frac{d(\sin x)}{(1-\sin^2 x)(\sin x)^{2/3}}=\left|t=\sin x\right|=\int\frac{dt}{(1-t^2)\,t^{2/3}}=$$

$$=\int t^{-2/3}\cdot(1-t^2)^{-1}dt=\left|\begin{array}{l}z=\sqrt[3]{t}\\ t=z^3\\ dt=3z^2dz\end{array}\right|\quad \begin{array}{l}dt=\frac{1}{3}z^{-\frac{2}{3}}dz\\ m=-\frac{2}{3},\ a=1\\ n=2,\ b=-1\\ p=-1\end{array}$$

$$\int x^m(a+bx^n)^p dx$$

$$=\int(z^3)^{-\frac{2}{3}}(1-(z^3)^2)^{-1}\cdot 3z^2\,dz=$$

$$=3\int\frac{dz}{1-z^6}=\left|\frac{1}{1-z^6}=\frac{1}{(1-z^3)(1+z^3)}=\frac{1}{(1-z)(1+z+z^2)(1+z)(1-z+z^2)}\right.$$

$$=\frac{A=1/6}{1-z}+\frac{Cz+D}{1+z+z^2}+\frac{B=1/6}{1+z}+\frac{Ez+F}{1-z+z^2}$$

$$\int\frac{dx}{\sqrt{\sin^3 x\cos^5 x}}=\left|\begin{array}{ll}t=\sin^2 x & \cos x=\sqrt{1-\sin^2 x}=\sqrt{1-t}=(1-t)^{\frac{1}{2}}\\ \sin x=t^{\frac{1}{2}} & x=\arcsin\sqrt{t}\ ;\ dx=\frac{1}{\sqrt{1-t}}\cdot\frac{1}{2\sqrt{t}}dt=\end{array}\right.$$

$$=(1-t)^{-\frac{1}{2}}\frac{1}{2}t^{-\frac{1}{2}}dt$$

$$\mu=-\frac{3}{2},\ \nu=-\frac{5}{2}$$

$$\mu+\nu=-4\text{ — парна}$$

$$=\int\left(t^{\frac{1}{2}}\right)^{-\frac{3}{2}}\cdot\left((1-t)^{\frac{1}{2}}\right)^{-\frac{5}{2}}\cdot(1-t)^{-\frac{1}{2}}\cdot\frac{1}{2}\times$$

$$\times\ t^{-\frac{1}{2}}dt=\frac{1}{2}\int t^{-\frac{5}{4}}(1-t)^{-\frac{7}{4}}dt=\left|\begin{array}{l}m=-\frac{5}{4},\ p=-\frac{7}{4}\ \ a=1\ \ b=-1\\ n=1\qquad \frac{m+1}{n}=\frac{-5/4+1}{1}=-\frac{1}{4}\ ;\ \frac{m+1}{n}+p=\\ dt=-\frac{4z^3}{(z^4+1)^2}dz\end{array}\right.$$

$$-\frac{1}{4}+\left(-\frac{7}{4}\right)=-2\in\mathbb{Z}$$

$$z=\sqrt[4]{1\cdot t^{-1}-1}\qquad t=\frac{1}{z^4+1}$$

$$z^4+1=t^{-1}\qquad 1-t=\frac{z^4}{z^4+1}$$

$$\frac{1}{2}\int(z^4+1)^{\frac{5}{4}}\left(\frac{z^4+1}{z^4}\right)^{\frac{7}{4}}\frac{-4z^3\,dz}{(z^4+1)^2}=-2\int(z^4+1)\cdot\frac{1}{z^4}dz=$$

$$=-2\left(\frac{z^{-3}}{-3}\right)+C$$

BUROMAX
Максимум якісної канцелярії