

**Презентація до навчальної дисципліни  
«Прикладна теорія катастроф у інженерії»**

## Catastrophe Theory



Vladimir I.  
ARNOLD



Rene  
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Christopher  
ZEEMANN

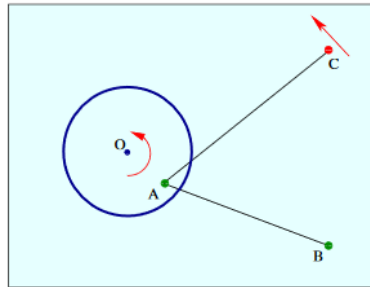
Originated by Vladimir I. Arnold, Rene Thom, and Christopher Zeemann in the 1960-70s,

**Catastrophe Theory studies and classifies phenomena characterized by sudden shifts in behavior arising from small changes in circumstances.**

Small changes in certain parameters of a nonlinear system can cause equilibria to appear or disappear, or to change from attracting to repelling and vice versa, leading to large and sudden changes of the behaviour of the system. Catastrophe theory has been applied to a number of different physical, chemical, biological, and even social phenomena.

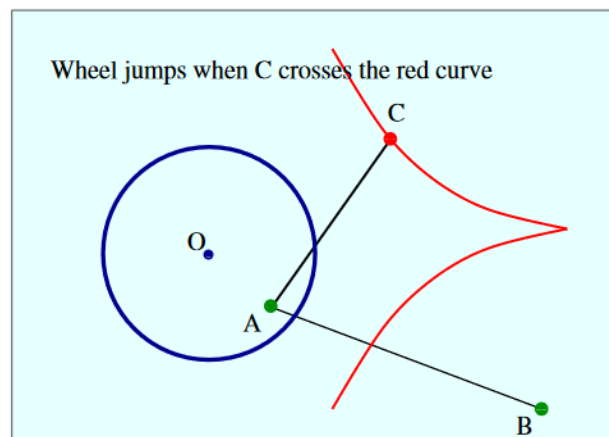
## Zeemann machine

This device, invented by the mathematician Christopher Zeeman consists of a wheel which is tethered by an elastic to a fixed point  $B$  in its plane. The control input to the system is another elastic attached to the same point  $A$  as the first and roughly of the same length. The other end  $C$  of the elastic can be moved about an area diametrically opposite to the fixed point.



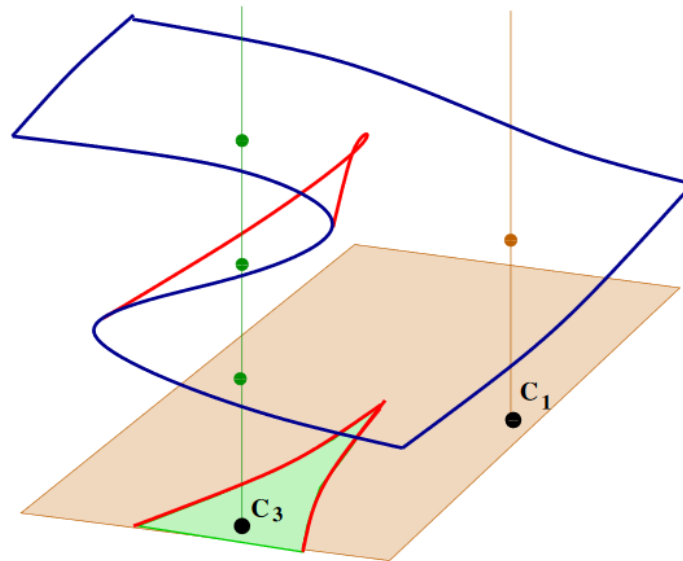
The experiment shows that:

- To each position of  $C$  there correspond 1 or 3 equilibrium points of the system.
- If we slowly move  $C$  then usually the wheel rotates slowly (this means that the equilibrium point slowly changes), however sometimes the wheel (the equilibrium point) suddenly jumps!
  - The positions of  $C$  where the equilibrium point jumps lie on a cusp.

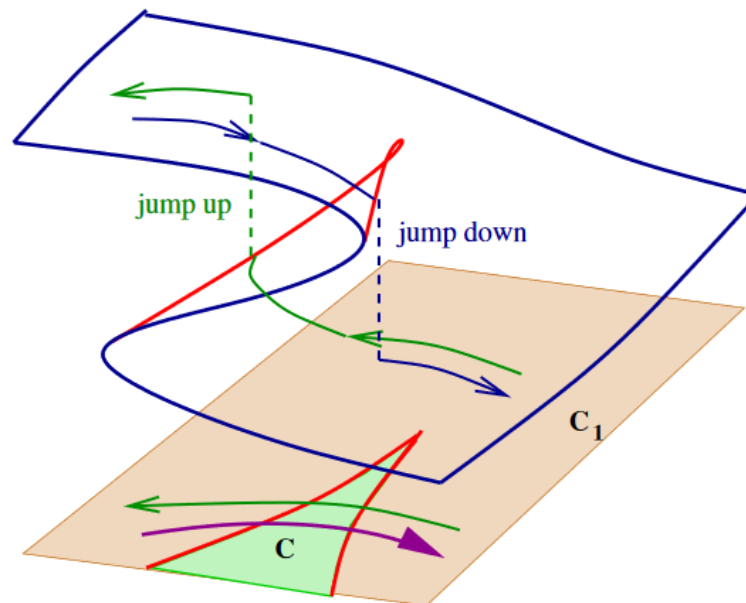


## Explanation of experimental phenomena.

1. The number of equilibrium points depends of the position of  $C$ .

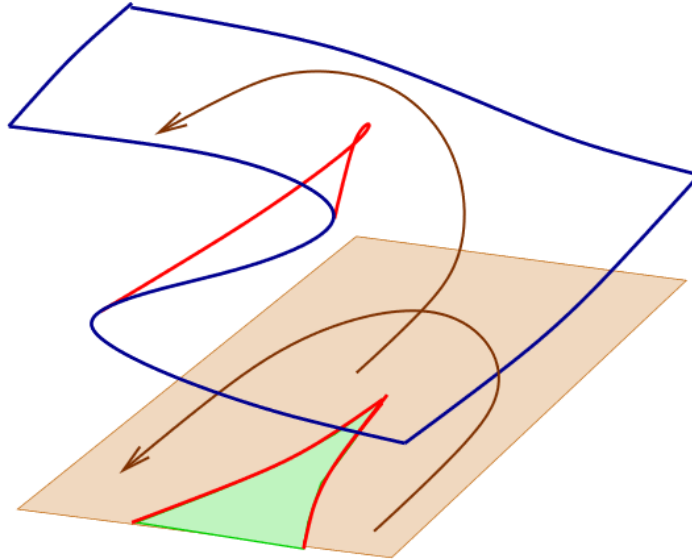


2. The equilibrium point jumps when  $C$  crosses the cusp.



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3. One can avoid the jump (the catastrophe)



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### **The mathematical formulation.**

We are given a system with one degree of freedom (one-dimensional configuration space) and  $m$ -dimensional control space.

$t$  is a coordinate in the configuration space (a parameter of the system);

$u_1, u_2, \dots, u_m$  are coordinates in the control space (control parameters);

$U(t, u_1, \dots, u_m)$  is the "potential energy" of the system;

**The main problem: How does an equilibrium point of the system behave when we vary control parameters?**

## Chemistry

### Van der Waals Equation of State

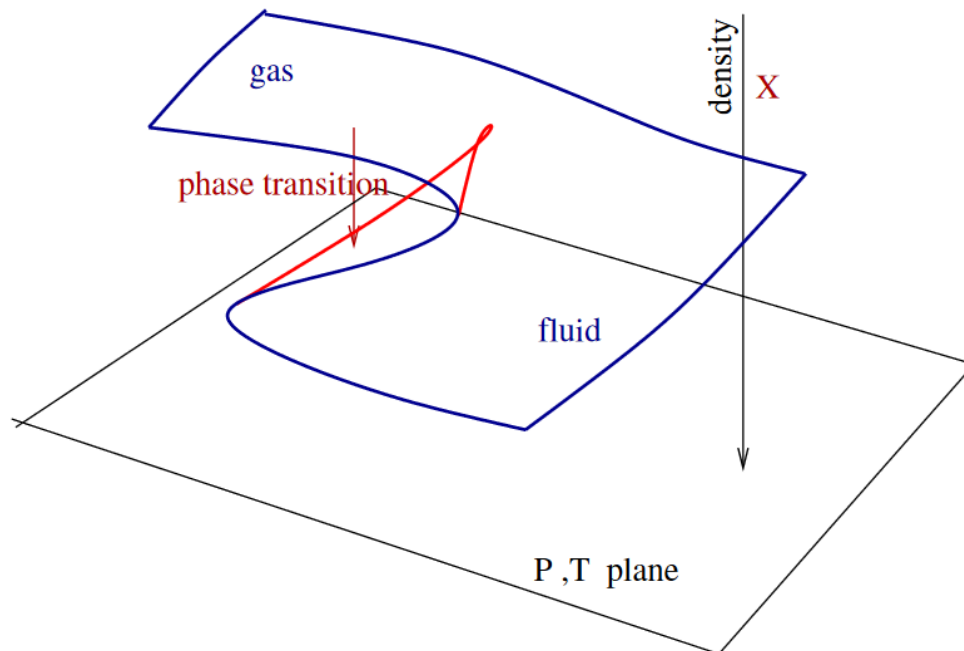
$$\left(P + \frac{\alpha}{V^2}\right)(V - \beta) = RT$$

Here  $P$  is the gas pressure,  $V$  is the gas volume, and  $T$  is the gas temperature, and  $\alpha$ ,  $\beta$  are parameters depending on the gas molecule properties.

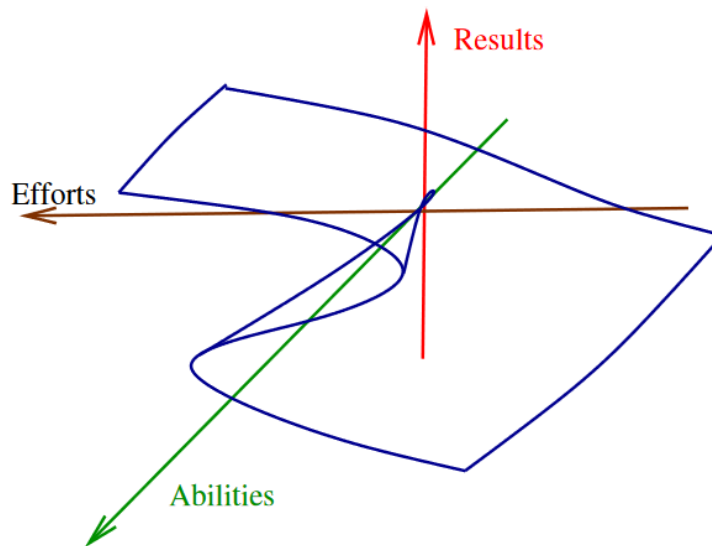
Set  $X = \frac{1}{V}$ , then after a linear change of coordinates we appear at the equation

$$X^3 + aX + b = 0,$$

where  $a$ ,  $b$  are linearly expressed in terms of  $P$ ,  $T$ .



## Psychology



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